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# Risk-neutralizing a loss distribution: Pricing the FDIC's reinsurance risk *

**Abstract**

This paper proposes a pricing model for the FDIC's reinsurance risk. We derive a closed-form Weibull call option pricing model to price a call-spread a reinsurer might sell to the FDIC. To obtain the risk-neutral loss-density necessary to price this call spread we risk-neutralize a Weibull distributed FDIC annual losses by a tilting coefficient estimated from the traded call options on the BKX index. An application of the proposed approach yield reasonable reinsurance prices and also shows that tilting is embedded in the FDIC's risk-based insurance premiums.

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# 1 Introduction

In 1991, the Federal Deposit Insurance Corporation Act authorized the Federal Deposit Insurance Corporation (FDIC), "to obtain private reinsurance covering not more than 10 percent of any loss the Corporation incurs with respect to an insured depository institution"($12 U.S.C.A$ $1817(b)(1)(B)$). Such authorization allows the FDIC to enter into financial contracts with the private sector that price and share bank default risk. Recently, the Options Paper produced by the FDIC (FDIC, 2000) view reinsurance as one way "to use market information to differentiate risks without imposing a particular funding structure on insured institutions." Given the significance of such a possible dramatic move toward market pricing of bank risk, it becomes essential to understand the elements of the reinsurance pricing, where a reinsurer provides excess-of-loss coverage to the FDIC. This paper responds to such a need and develops a pricing model for excess-of-loss reinsurance risk. However, we should note at the outset that our findings should not be construed one way or another as a position paper for or against a private sector reinsurance arrangement for the FDIC.

From a contingent claims point of view an excess-of-loss reinsurance contract represents a portfolio of call options written on the aggregate loss level of the FDIC. Effectively, the reinsurer sells the FDIC a call-spread where the reinsurer will have to cover losses above a strike level but its commitment is capped by a stated coverage level. In this paper, we obtain a closed form pricing expression for the call-spread under the assumption that the underlying FDIC losses follow a Weibull distribution, which is in the family of extreme-value distributions. We assume that the reinsurer uses the risk neutral distribution of losses to price the reinsurance risk. However, the reinsurer only observes the FDIC's historical (statistical) loss distribution. We show that the reinsurer can obtain the risk neutral density by exponentially tilting the FDIC's statistical loss distribution by a coefficient obtained from the traded options markets. We explain how this exponential tilt may be seen as an approximation to a more general tilt that scales the cash flows by the conditional expectation of the market pricing kernel.

The use of exponential tilts has a long history in finance. For one, it is now recognized that Black-Merton-Scholes option pricing results from applying an exponential tilt applied to the underlying Brownian motion (Duffie, 1992). In the context of the Black-Merton-Scholes complete markets model this exponential tilt is in fact the unique complete markets solution. The idea has been subsequently used in a variety of incomplete markets contexts including Heston (1993), where the risk in the Brownian motion driving the volatility is priced by exponential tilting. More generally for a diffusion filtration it is well known (Karatzas and Shreve, 1991) that all measure changes are locally exponential tilts of the underlying Brownian motions. The method has been employed in the term structure literature. (see for example, Heath, Jarrow, and Morton,1992). In models with jumps, Naik and Lee (1990) use exponential tilts by employing constant relative risk aversion utility functions.

Our approach can also be related to recent literature estimating the risk-neutral and statistical densities to make inferences about the implied risk-aversion coefficients (see for example, Jackwerth, 2000; Ait-Sahalia and Lo, 2000; Ait-Sahalia, Wang, and Yared, 2001; Coutant, 2001; Bakshi, Kapadia, and Madan (2003); Bliss and Panigirtzoglou, 2004). Collectively, this literature focuses on the entire distribution of the underlying assets values. However, catastrophe reinsurance contracts, such as the reinsurance of the FDIC's losses, have zero payoff in the center of distributions and pricing these instruments requires focusing on the tail of the distribution of the underlying asset values. Hence, we add to this literature by utilizing extreme value theory that characterizes the tail distributions of positive random variables like loss levels and estimate the tilting applicable to the tail events rather than the entire distribution of the outcomes.

To estimate the statistical distribution of the FDIC's annual losses on bank failures, we follow Madan and Unal (2003), who analyze the distributional properties of the losses in bank failures of the 1986-2000 period. They show that the two-parameter Weibull distribution best characterizes the FDIC's loss distribution in about 1,300 bank failures. Lucas, Llaassen, Spreij, and Straetmans

(2001) also propose the use of Weibull density from a theoretical perspective to capture the distribution of loss rates. We estimate the statistical parameters of the Weibull distribution for annual losses incurred by the FDIC during 1986-2000.

To infer the applicable tilt coefficient in the traded options markets we examine the prices of deep out-of-the-money calls on bank equity (BKX) index and estimate the implied risk neutral distribution in this market. We next estimate the statistical Weibull distribution of the BKX index and infer the constant tilting coefficient implied by the statistical and risk neutral distributions. We risk-neutralize the FDIC's statistical loss distribution with the level of tilting implied in the BKX index. Our findings provide a range of values for the FDIC's reinsurance risk from which one can assess the reasonableness of the reinsurance prices.

We compare our estimated prices with those of MMC Enterprise Risk (MMC). In a report submitted to the FDIC, MMC provides two rough price estimates FDIC might have to pay to a private insurer to purchase a call-spread (MMC, 2001, p. 21). Our calculations show that MMC estimates are in the vicinity of our price-estimates thus reflect a risk neutral pricing rather than a statistical one. In another application of our findings, we estimate that the FDIC needs to charge the banking system \$4.3 billion in aggregate insurance premium for loss coverage of \$26.56 billion. This aggregate insurance cost represents 22.4 cents on \$100 insured deposits at the level of \$1.9 trillion insured deposits. Given the proximity of this estimate to the effective insurance premiums assessed by the FDIC, we assert that FDIC is implicitly tilting the statistical distribution of its losses with a tilt coefficient that is close to the one observed in the traded options market.

The paper is organized as follows. Section 2 presents the underlying framework for our analysis. Section 3 derives the Weibull option pricing model and section 4 estimates the implied risk aversion in the options market. Section 5 shows the application of reinsurance pricing. Section 6 concludes the paper.

## 2 Pricing the reinsurance contract

### 2.1 The Weibull call option pricing model

An excess-of-loss reinsurance contract is a portfolio of call options written on the aggregate loss level, $L$, of the FDIC. The first call option is written by the reinsurer on the FDIC's aggregate loss level at a strike $K$. As the buyer of this call, the FDIC incurs losses up to $K$ but receives from the reinsurer the loss amount $L$ exceeding the strike $K$. However, the reinsurer's coverage of losses above the strike is not unlimited and payments are capped at $K + B$, where $B$ is the the stated coverage level. This condition implies that the FDIC simultaneously sells the reinsurer a call option struck at $K + B$. This second call caps the reinsurer's payout at the coverage level $B$. Taken together, this portfolio of two call options implies that the reinsurer sells the FDIC a call-spread.

For a coverage level of $B$ the loss contingent payoff to the call-spread, $CF(L)$, at year end can be expressed as:

$$\begin{aligned} CF(L) &= Min\left(Max\left(L - K, 0\right), B\right) \qquad (1) \\ &= Max(L - K, 0) - Max(L - (K + B), 0) \end{aligned}$$

The first call option in Equation (1) represents the call option the reinsurer sells the FDIC written on the FDIC's aggregate loss level, $L$, at a strike $K$. The second option ensures that the reinsurer's coverage of losses above the strike is capped at $K + B$.

We suppose that the quoted prices for claims of this type are free of arbitrage and hence assume the existence of a risk neutral probability of a loss arrival $\lambda$ and a risk neutral conditional density $q(L)$, of the loss levels $L$. Specifically, the price of this call-spread $w$, given an annual continuously compounded risk-free interest rate of $r$ is given by

$$w = \lambda e^{-r} \int_0^\infty CF(L) q(L) dL. \qquad (2)$$

For the particular application made in this paper we note that the FDIC has always incurred some level of loss in each year since 1980. We assume therefore that the statistical probability of a loss arrival is one. By the equivalence of risk neutral probabilities to the underlying statistical probabilities it follows that one may assume the risk neutral $\lambda$ is also one.

To price the call spread, the reinsurer then just needs to identify a relevant risk-neutral probability distribution for annual loss levels, $q(L)$. This density describes the current market price of loss contingent bonds that pay one-dollar face in a year on the contingency that particular loss levels are attained. The focus of any reinsurance contract is on the tail of this distribution of loss levels. Essentially, the critical question is to have an adequate description of the tail behavior as the call spread contracts of interest have a zero payoff at low loss levels. For models of the tail we turn to extreme value theory that characterizes the tail distributions of positive random variables like loss levels.

There are basically three parametric classes of distributions that characterize tail behavior. These are the Frechet, Gumbell and Weibull (Embrechts, Kluppellberg, and Mikosch (1997)). We note that of the three, the Weibull describes the limiting behavior of scaled maximal losses drawn from random variables with an upper bound. In the present context the potential loss levels are bounded above by the size of assets in place and hence such a distribution might be the right choice. Indeed, Lucas, Llaassen, Spreij, and Straetmans (2001) use Weibull to describe extreme tail behavior of credit losses in terms of portfolio characteristics. Additionally, an investigation conducted by Madan and Unal (2003) into densities describing FDIC loss distributions shows that of these three the Weibull distribution provides the best fit to this data.[1] These considerations lead us to proceed with the Weibull model for the purposes of the present study.

The specific functional form for the Weibull density, $g(L; c, a)$ with parameters $c$ and $a$ is given

[1] Weibull, Frechet, Beta, and Logit normal distibutions are fit to the loss experience of the FDIC. For a more detailed discussion of the relative properties of these densities in the context of financial loss modeling we refer the reader to Madan and Unal (2003).

by

$$g(L;c,a) = \exp\left(-\left(\frac{L}{c}\right)^a\right)\frac{aL^{a-1}}{c^a} \tag{3}$$

with mean $\mu$ and standard deviation $\sigma$

$$\mu = c\Gamma\left(1+\frac{1}{a}\right) \tag{4}$$

$$\sigma = c\sqrt{\Gamma\left(1+\frac{2}{a}\right) - \Gamma\left(1+\frac{1}{a}\right)^2} \tag{5}$$

where $\Gamma(x)$ is the gamma function.

The parameter $c$ is a scaling parameter and $a$ is called the shape parameter. The value of $a$ determines the relative fatness of the tail of the distribution, with higher values of $a$ leading to thinner tails. We see from Equations (4) and (5) that the coefficient of variation is determined by the parameter $a$.

We next derive in Proposition 1 the closed-form expression for the price of the call option $(Max(L-X,0)$ with strike $X$ written on the loss level $L$ , which is distributed Weibull.

**Proposition 1** *The value of the Weibull call option, with parameters c and a, written on the loss level L with strike X is given by*

$$C = e^{-r}\left[L^*W_1 - XW_2\right]$$

$$\text{where } L^* = c\Gamma\left(1+\frac{1}{a}\right) \tag{6}$$

$$W_1 = 1 - gammainc\left(\left(\frac{x}{c}\right)^a, 1+\frac{1}{a}\right) \tag{7}$$

$$W_2 = \exp\left(-\left(\frac{X}{c}\right)^a\right) \tag{8}$$

*and $L^*$is the expected loss level under the risk-neutral measure, $\Gamma$ and gammainc are the gamma and incomplete gamma functions.*

*Proof in the Appendix*

We note the Weibull call option formula has a similar structure to the Black-Scholes option pricing formula. The present value of the strike is multiplied by the Weibull risk neutral probability that the call is in the money. $L^*$ is the risk neutral expected loss level and this is multiplied by $W_1$, which is the probability of the call being in the money under a suitably adjusted measure. It follows that a reinsurer can obtain the theoretical expression for the call-spread, given in Equation (2), as the difference between the two call options with strikes $K$ and $K+B$ written on loss levels that is Weibull distributed.

## 2.2 Risk neutralization strategy

The reinsurer, however, only observes the historical loss experience of the FDIC and must then estimate the parameters of the risk-neutral Weibull distribution to obtain a price quote. For a perspective we turn to the extensive literature on the reinsurance of catastrophic disaster losses. This literature is divided in the way it risk-neutralizes an estimated statistical distribution. As elegantly explained in Cummins, Lewis, and Phillips (1999) two approaches exist in pricing catastrophic insurance pricing. The first approach employs utility functions of risk averse agents to construct the risk-neutralized density and this is termed the actuarial approach. In contrast, the financial approach models the determinants of marginal utility of a representative agent, decomposing the risks into a systematic and diversifiable component. For diversifiable risks it can be shown that the risk neutral and statistical density coincide with no adjustments being necessary. For example, Cummins, Lewis, and Phillips (1999) suppose that catastrophic losses are diversifiable and employ the statistical distribution directly in pricing.

However, diversification is typically accessible via a large number of draws on independent and comparable events, that in particular do not involve dominating events. This is questionable for contexts dealing with events that are extremal and dominating by design. Thus, we address the issue of how to specifically risk neutralize the statistical density in the absence of access to appropriate diversification. In contrast to adopting a specific utility based approach we investigate the relationship between the risk neutral and statistical densities in active options markets where both densities may be adequately estimated. Indeed, Bakshi, Kapadia, and Madan (2003) show that the risk neutral density in options markets is related to the statistical one by a renormalized exponential tilt. Furthermore, we note that in many insurance applications risk-neutral and statistical distributions are related by what is called the Esscher transform that exponentially tilts the statistical distribution to determine the risk neutral distribution (Esscher (1932), Sondermann (1991), and Gerber and Shiu (1996)).

Hence, our approach to identify $q(L)$ is to assume a statistical distribution for the FDIC's historical loss levels, $p(L)$, and tilt this statistical distribution by an exponential tilt coefficient, $\alpha$, and renormalizing it to obtain the risk-neutral density as follows:

$$q(L) = \frac{e^{\alpha L} p(L)}{\int_0^\infty e^{\alpha L} p(L) dL} \tag{9}$$

To determine the coefficient of exponential tilting from the traded options market, we assume that the reinsurer estimates $p(S)$ from the time series of asset returns and $q(S)$ from the prices of options that are written on these assets. Hence, once the specific densities for the distributions $p(S)$ and $q(S)$ that appropriately characterize the underlying are specified for the options market, the reinsurer can estimate the implied tilting coefficient, $\alpha$, between the two using Equation (9) as follows:

$$\log\left(\frac{q(S)}{p(S)}\right) = -\log\left(\int_0^\infty e^{\alpha S} p(S) dS\right) + \alpha S, \tag{10}$$

Note that we can construct a linear approximation to the logarithm of the ratio of the two densities by employing a least squares curve fitting scheme in which the error is a deterministic error of approximation in functional forms. This estimation can be conducted by running the following regression equation:

$$\log\left(\frac{q(S)}{p(S)}\right) = k + \alpha S + \epsilon. \tag{11}$$

Note that in Equation (11) there are no statistical dimensions involved as the error term $\epsilon$ is nonrandom, and $k$, $\alpha$ are the intercept and slope of the linear approximation. Thus, once the estimates of $k$ and $\alpha$ are obtained the reinsurer can estimate the risk neutral density of the losses,

$q(L)$, given $p(L)$ as follows

$$q(L) = \exp(k + \alpha L)p(L) \tag{12}$$

### 2.3 Exponential tilts and asset pricing

The strategy of exponential tilting is consistent with a number of theoretical results on asset pricing in economics. When we have a complete markets equilibrium in which the pricing kernel is uniquely determined, we may view exponential tilting as a local approximation to a more general tilt that is determined by the conditional expectation of the pricing kernel. The variations in the specific tilts across the range of state space of the asset price and across assets then reflect local covariations between the asset and the kernel. Somewhat more formally, the price, $w$, of claim to a state contingent cash flow $c(\omega)$ can be written as the discounted at the risk-free rate of return $(r)$ of the expected cash flow, where expectation is taken at the risk-neutral measure, $E^Q$

$$w = e^{-r}E^Q\left[c(\omega)\right] \tag{13}$$

Alternatively, the expectation can be taken at the statistical measure, $E^P$as follows

$$w = e^{-r}E^P\left[\Lambda(\omega)c(\omega)\right] \tag{14}$$

where $\Lambda(\omega)$ is the change of measure density. Conditioning on the price of the underlying asset $S$ we may write

$$w = e^{-r}E^P\left[E^P\left[\Lambda(\omega)|S\right]c(S)\right] \tag{15}$$

where we have supposed for simplicity that the claim is contingent only on the value of $S$. Defining by

$$g(S) = E^P\left[\Lambda(\omega)|S\right] \tag{16}$$

we see that the market price for a claim is an expected value of the tilted cash flow, tilted by $g(S)$. Further, $g$ is a positive function of the real valued variable $S$ and the mixture of exponentials in $x$

is a spanning set of functions for all potential tilt functions $g$. For a local analysis of the behavior in a part of the tail, the use of a single exponential is adequate and this can be checked by the quality of the regression of $\ln(q/p)$ on $S$ as given in Equation (11). Note that in Equation (11) one is merely constructing a local linear approximation to $\ln(g)$ in $S$ . If the specific asset risk is not priced in the region under study, then would we expect to observe $k = \alpha = 0$ in equation (11).

From an incomplete markets point of view the pricing kernel to be used is no longer uniquely determined and is individual specific as well. In this case, tilt variations also incorporate differences in the risk aversions of participants as well as informational variations on the distribution of the risks involved, to the extent that the scaled Weibull model is an approximation to the true and richer statistical model. For a specific connection between equation (9) and utility theory the reader is referred to Appendix 7.2 that relates exponential tilts to a specific utility function. The coefficient of exponential tilting can then be related to a constant absolute risk aversion coefficient of an agent facing the losses.

The next section provides the details for estimating the tilt coefficient $\alpha$. This exercise is followed by the estimation of the statistical loss density for the FDIC. Finally, we tilt this density to obtain the risk neutral density and price the call spread.

# 3 Weibull implied exponential tilt in the options market

## 3.1 Estimation approach

To approximate the appropriate tilt coefficient we assume that the reinsurer examines market risk preferences on an asset that best reflects the aggregate bank risk. One proxy for such an asset is the PHLX / KBW Bank Index (BKX). BKX is a capitalization-weighted index composed of 24 geographically diverse stocks representing national money center banks and leading regional institutions. The index is evaluated annually by Keefe, Bruyette & Woods to assure that it represents the banking industry. The index was initiated on October 21, 1991 and options started trading on September 21, 1992.

We view the reinsurer as basically a financial firm that is positively correlated with the banking sector and is well funded. Such a reinsurer is not on the buy side of the bank index out-of-the-money put market and probably not on the sell side as well. The incentives for extra premia may be expected from a short position in out-of-the-money BKX calls and these considerations lead us to use the upside call tilting coefficient as an appropriate level of risk aversion.

We assess the degree of exponential tilting that occurs in pricing out-of-the-money equity call options at the 1, 5, and 10 percent risk levels. We do this analysis by estimating the statistical and risk neutral densities in the upper tail of the returns, denoted by $p(S)$, and $q(S)$ respectively, and estimate the regression equation given in equation (11) for values of $S$ in the upper tail of the statistical distribution.

To maintain consistency with the FDIC's loss distribution, we choose the Weibull as the functional forms for these densities. We focus attention on returns over a prespecified horizon and let $S$ be the final stock price while $S_0$ denotes the initial stock price.

We define the excess return

$$R = \frac{S}{S_0} \tag{17}$$

that is a positive random variable for which the Weibull distribution is an appropriate extreme value density reflecting finite moments of all orders. For both the statistical and risk neutral density we suppose the density of $R$ has the Weibull form with parameters $c$ and $a$ :

$$f(R) = \exp\left(-\left(\frac{R-1}{c}\right)^a\right)\frac{a(R-1)^{a-1}}{c^a} \tag{18}$$

We estimate from time series data on daily returns, using 25% of the largest and smallest returns the statistical parameters *cust* and *aust* representing the statistical upside $c$ and $a$ values.

For a comparison with the risk neutral density we have to construct returns at the option maturity from the estimated daily return distribution. However, in making a comparison with risk neutral densities there is a horizon mismatch, as risk neutral densities are observed over much longer horizons than a single day. This mismatch has hindered the research agenda on these questions for some time now, as it is not clear how one should construct longer horizon returns from good estimates of short horizon likelihoods. There is a temptation to assert that forward daily returns are sampled from independent distributions that are identical to the density for the current daily return. However, such a procedure runs against the basic intuition that daily returns are state contingent entities, where the current state is much clearer than the possibilities open for the forward state. One could attempt to identify by time series or vector autoregression methods, a high dimensional Markov representation for the data generation mechanism associated with asset returns, but this is likely to take us astray into a host of time series and econometric issues.

Instead, we recognize the nature of the basic intuition that uncertainties pertaining to the distant future are rising as we contemplate forward returns at the current time by using a scaling hypothesis. Under this hypothesis we model the return at a horizon of $N$ days as having the distribution of $\sqrt{N}$ times the daily return distribution, or define the return over $N$ days, $R_N$ to be in law

$$R_N - 1 \overset{law}{=} \sqrt{N}(R-1) \tag{19}$$

The variance then grows linearly with $N$ as it would were we to add independent and identically distributed, but unlike the situation with addition of independent random variables, skewness and excess kurtosis remain constant in $N$. For the case of adding i.i.d. variables, skewness falls like $\frac{1}{\sqrt{N}}$ and excess kurtosis falls like $\frac{1}{N}$ as shown in Konikov and Madan (2002). In the sense of the higher moments, the uncertainty is maintained at a higher level than would be the case with summing independent and identically distributed random variables.[2]

[2] Alternatively, one may appeal to the work of Sato (1999) who shows that the class of all limit laws of arbitrarily scaled sums of independent but not necessarily identical random variables are the laws at unit time of a scaled process of independent and generally inhomogeneous increments. This observation makes such processes relevant to the modeling of financial returns, that may easily be seen as the limit of the sum of a large number of independent effects.

Thus, it follows that the Weibull density for $R_N$ is

$$f(R_N) = \exp\left(-\left(\frac{R_N - 1}{c\sqrt{N}}\right)^a\right)\frac{a(R_N - 1)^{a-1}}{(c\sqrt{N})^a} \tag{20}$$

and with parameters $c\sqrt{N}, a$.

The corresponding risk neutral values are denoted *curn* and *aurn* and are estimated by calibrating the model prices developed under the specific density to the prices of the out-of-the-money call options. For this task, following Proposition 2, we develop the Weibull call option pricing formula using the Weibull density.

For a call option of maturity $t$ the call option value, $cv$,

$$cv = e^{-rt}\int_{\frac{K}{S_0}}^{\infty}(S_0R - K)\exp\left(-\left(\frac{R-1}{c}\right)^a\right)\frac{a(R-1)^{a-1}}{c^a}dR. \tag{21}$$

Note that in equation (21), we do not impose the condition that the discounted stock price is the current stock price as we do not assert that the Weibull density applies for all levels of the stock price, but only applies in the upper right tail where the specified calls are in the money. Traditional option pricing models model the entire distribution of the underlying asset and hence must enforce the spot forward arbitrage condition requiring that

$$S_0e^{rt} = \int_0^{\infty} S_tq(S_t)dS_t \tag{22}$$

or that the financed stock purchase have zero price. Since we focus on the Weibull model for just the tail of the distribution, and use it to price out of the money calls on the up side, we do not have a condition integrating across the entire of stock prices. In fact, we impose no distributional hypothesis at all, in the center of the distribution, or the near money density.

Performing the requisite integration in equation (21) we obtain that

$$\begin{aligned} cv &= e^{-rt}(S_0 - K)\exp\left(-\left(\frac{\frac{K}{S_0} - 1}{c}\right)^a\right) + \\ &\quad S_0ce^{-rt}\Gamma(1 + \frac{1}{a})\left(1 - gammainc\left(\left(\frac{\frac{K}{S_0} - 1}{c}\right)^a, 1 + \frac{1}{a}\right)\right) \end{aligned} \tag{23}$$

Hence, the risk-neutral parameters $curn$ and $aurn$ can be estimated by calibrating equation (23) to the out-of-the-money call prices struck at the top five strikes trading in the market. For these strikes we use a maturity of around 2 months.

### 3.2 Results on the BKX Index

We use time series data on the Bank Index (BKX) for 1500 days ending on September 28, 2001 to obtain the statistical distribution and data on index options for every second wednesday of each month over the year beginning in October 2000 and ending in September 2001 to estimate the risk-neutral distribution. To estimate the statistical parameters of the Weibull density we first compute the upside returns as described in the previous section. We sort these returns and extract the top 25% of returns. The Weibull model is estimated by maximum likelihood on large positive returns to yield the statistical parameters for the BKX index. The estimated parameters are $cust = 0.0345$ and $aust = 2.5324$.

The risk neutral parameters are estimated by calibrating model prices (equation (23)) to the call option prices with maturity of around two months with the five largest strikes trading in the market for this maturity. The calibration is done for one day in each month from October 2000 to September 2001. The results are presented in Table 1 for the BKX index, where the average $curn = 0.0645$ and $aurn = 0.9413$.

Next, the regression equation (11) is estimated where the logarithm of the ratio of the risk neutral density to the scaled statistical density regressed on the price level in the range between 1% to 0.01% return levels in two months. The resulting slope coefficients are the associated levels of exponential tilting on the upper tail of the return distribution. The results are presented in Table 1 along with the mean levels of tilting for the BKX. We observe that the mean levels of tilting to losses on the up side in BKX is 0.1739. Hence, we propose to tilt the statistical distribution of the FDIC's losses by this coefficient to obtain the risk neutral distribution.

## 3.3 Tilt coefficients for individual banks

For a perspective on the level of the tilt coefficient estimated for the **BKX**, we present similar calculations for a sample of 18 bank holding companies. We estimate by maximum likelihood the statistical parameters of the Weibull density for each individual bank using the top 25% of returns of preceding 1000 trading days of every second wednesday of the month from September 1998 to December 2002. The risk neutral parameters are estimated by calibrating model prices (equation (23) to the call option prices on every second wedesday of the month with strikes at least 10% out of the money and maturity between 25 and 50 days. Estimations are performed with at least three options and in the case when there are less than three option prices, we denote the week as invalid. Therefore the number of valid weeks out of a possible 112 possible weeks vary among individual bank estimations. Once the statistical and risk neutral densities are obtained tilt coefficient for each bank is estimated by utilizing the regression equation (11).

Table 2 reports mean and median values of the estimated tilt coefficient for each bank in columns 1 and 2, respectively. Column 3 shows number of weeks when estimation could be undertaken given the available data points. Column 4 shows the number of options used on average to estimate the risk neutral distribution. From column 1, we observe that the level of individual tilt coefficients ranges between .14 and .31. Such variation is expected to the extent that these coefficients reflect the risk attributes of the individual names. To have a rough assessment of such an expectation we report in column 5 of Table 2 the average credit spreads observed weekly during January 2000 through May 2001.[3] We regress these credit spreads on the estimated tilt coefficients and observe that the slope coefficient is positive and significant at the 1% significance level with $R^2 = 0.33$ and the correlation coefficient is 0.58.Thus, these preliminary findings provide some support that our estimated tilt coefficients reasonably capture bank risk.

[3] We thank Gerry Hanweck for providing us the spread data.

# 4 Calculating the price of the FDIC's aggregate loss risk

## 4.1 Statistical parameter estimation

Consistent with the above approach we should estimate by maximum likelihood the statistical parameters of the Weibull distribution applicable to the loss experience of the FDIC. However, the frequency of available data poses a special difficulty. Our focus is to capture the distribution of annual losses and we have only 15 years of annual loss data covering 1986-2000. The aggregate annual loss levels of the FDIC is displayed in Table 3. Although we could increase sample observations by including years dating back to 1930s we find this manner of expanding the sample undesirable because these dated periods are not reflective of risks faced by the today's FDIC. Thus, application of maximum likelihood approach is not appropriate for such a small number of observations.

We resolve this difficulty of estimating the statistical parameters $c$ and $a$, by using the sample mean and standard deviation of the FDIC's annual loss experience for $\mu$ and $\sigma$ and invert the following:

$$1+\frac{\sigma^2}{\mu^2}=\frac{\Gamma\left(1+\frac{2}{a}\right)}{\Gamma\left(1+\frac{1}{a}\right)^2} \tag{24}$$

Equation (24) is derived from Equations (4) and (5). From Table 2 we observe that the annual mean and standard deviation of annual loss levels between 1986-2000 is $\mu$ = \$2.106 billion and $\sigma$ = \$2.497 billion, respectively. Substituting these values in Equation (24) and using method of moments, we obtain the statistical parameters for the Weibull distribution as $c = 1.9317$ and $a = 0.8472$.

## 4.2 The value of the call-spread

We can price the call spread now by using the tilt coefficient of 0.1739 to tilt the statistical distribution and obtain the risk-neutral distribution. We estimate the price of a call spread in the context of a reinsurance quote estimate given to the FDIC. Recently, the FDIC retained MMC

Enterprise Risk (MMC) to determine the feasibility and the costs of private sector reinsurance arrangements. In a report submitted to the FDIC, MMC provides two rough price estimates for reinsuring the aggregate annual losses of the FDIC (MMC, 2001, p. 21). The specific estimates are such that the annual premium on a \$2 billion coverage at a *one* basis point (less than one chance in 10,000) risk level is \$4 million. A second price estimate states that the annual premium on a higher risk level of *one* percentage point (one chance in 100) with \$0.5 billion dollar coverage is \$10 million. Although the strike levels are not specifically indicated in the report we can easily estimate the implied strikes given the parameters of the statistical Weibull density.

Note that the probability of a loss amount exceeding the strike is given by

$$P(L > K) = \theta = 1 - F(K) \tag{25}$$

For the Weibull cumulative distribution function,

$$F(K) = 1 - \exp\left[-\left(\frac{L}{c}\right)^a\right] \tag{26}$$

equation (25) is written as

$$-\log(\theta) = \left(\frac{K}{c}\right)^a \tag{27}$$

Hence, the strike is expressed as

$$K = c\,(-\log(\theta))^{\frac{1}{a}} \tag{28}$$

Substituting the estimates of $c$, $a$, and the risk level, $\theta$, in equation (28) we obtain $K_1 = \$11.72$ billion and $K_2 = \$26.56$ billion for the high risk and low risk cases, respectively. We note that these estimates of strike levels, implied by the quoted prices, can be considered reasonable because they are well within the current \$30 billion FDIC deposit insurance fund level.

To risk neutralize the statistical loss density of the FDIC we need to exponentially tilt it by the level observed in the pricing of $BKX$ options on the up side, 0.1739. Toward this end, we might use equation (12). However, exponentially tilting a Weibull $p(L)$ as in equation (12) does not yield a Weibull $q(L)$. As an alternative approach, we alter the risk neutral density such that the derivative of the logarithm of the risk neutral density accounts for the altered tilt. Specifically, using equation (9) the estimated tilting is basically the difference between the derivative of the logarithm of the risk neutral and statistical densities,

$$\frac{d\log q(L)}{dL} - \frac{d\log p(L)}{dL} = 0.1739$$

We evaluate the derivative of the logarithm of $p$ at the two strikes of 11.72 and 26.56 to be $-.3638$ and $-.2871$. This provides us two equations

$$\begin{aligned} \frac{d\log q(L)}{dL}|_{11.72} &= -0.1899 \\ \frac{d\log q(L)}{dL}|_{26.56} &= -0.1132 \end{aligned}$$

from which we can simultaneously solve for the parameters $c$ and $a$ of $q(L)$ to obtain $c = 1.0517$ and $a = 0.5050$. Using these values for the risk neutral parameters we price the two call-spreads to obtain the values

$$\begin{aligned} w(.01, .5) &= 16,120,000 \\ w(.0001, 2) &= 10,802,215 \end{aligned}$$

These prices compared with those of the MMC estimates appear to be overpriced by \$6 million in each case.

We can place these estimates in perspective by pricing the call-spread statistically. In other words, assuming risk neutrality we can use the statistical mean \$2.106 billion and standard deviation \$2.497 billion as our working Weibull distribution and estimate the actuarially fair prices. Under this assumption, using equation (6), the call-spread is valued statistically at \$4.5 million and \$150,000 for 1 % and 0.01 % risk levels, respectively. Note that the statistical prices establish the lower bound for the reinsurance price risk. Thus, we observe that MMC estimates reflect some level of tilting in pricing rather than assuming risk neutrality on the part of the reinsurer..

### 4.3 Pricing the aggregate coverage

The current assessment system used by the FDIC requires the FDIC to charge at least 23 cents per \$100 deposits if the mandated reserves to insured deposits, designated reserve ratio (DRR), is below 1.25%. The Deposit Insurance Funds Act of 1996 prohibits the FDIC from assessing depository institutions as long as DRR is above 1.25%. As of December 31, 2000, 92% of all insured institutions were not paying premiums for deposit insurance (FDIC, 2001).

Our estimates of the FDIC's risk neutral loss density can also be used to compute the aggregate premium that should be collected from the insured institutions. If we accept the degree of exponential tilting outlined above, then the risk neutral density for coverage up to \$26.56 billion is:

$$q(L) = \frac{e^{0.1739L} p(L : \mu = 2.106, \sigma = 2.497)}{\int_0^{26.56} e^{0.1739L} p(L : \mu = 2.106, \sigma = 2.497) dL}. \tag{29}$$

Note now that as the FDIC offers the random coverage level $L$ each year then the aggregate premium that should be collected at the 0.01% risk level from the insured institutions is the price of this coverage and this is given in forward terms by:

$$\Pi = \int_0^{26.56} L q(L) dL. \tag{30}$$

For the specific risk neutral distribution given in equation (29), we compute this integral at \$4.2764 billion. For the level of insured deposits around \$1909.9 billion this is a premium of 22.4 cents per \$100 deposits for the year 2001, which is quite comparable with the average deposit insurance premium charged by the FDIC when the insurance fund is below 1.25% of the insured deposits. In addition, this estimate of the aggregate deposit insurance premium is quite comparable with those of Cooperstein, Pennacchi, and Redburn (1995), who estimate the fair premium to be in the range of $23.8 - 24.9$ cents for years 2000 and 2001. We also estimate the average premium for the same coverage assuming tilt coefficients of 0.308 and 0.141. These are the highest and lowest coefficients estimated for the individual banks reported in Table 2. At these levels the insurance premiums are estimated to be 52.79 cents and 18.84 cents, respectively. In other words, we should expect the insurance premiums to vary as the risk levels and hence the tilt coefficients change over time and across banks.

Calculated statistically, the value of the integral in equation (30) is \$2.1032 billion. This value represents deposit insurance premiums of 11 cents per \$100 deposits. In other words, we can assert that the assessed deposit insurance premium is consistent with FDIC tilting the distribution of its historical loss experience.

To ensure that the level of the fund is a risk neutral martingale, 22.4 cents premium should be collected each year (assuming no change in statistical and the risk neutral distributions). However, statistically, the fund will have a positive expected cash flow and it is therefore expected to grow over time in line with the return commensurate with the insurance business it is engaged in (Pennacchi, 2000). The question does arise as to who gets the expected return from this activity. Although this questions begs in depth analysis, we can assert that if the fund is viewed as mutually owned by the insured institutions then the growth may be transferred to them in the form of reduced premiums and this could be the logic underlying the decision to reduce premiums to zero in certain growth situations.

# 5 Conclusion

Our findings can be summarized as follows. We envision the reinsurer to be facing the FDIC's statistical loss experience, which we estimate to be Weibull distributed with mean \$2.106 billion and standard deviation \$2.497 billion. We propose that the reinsurer risk-neutralizes this distribution by a tilt coefficient obtained from the traded call option prices on the BKX index. Application of the call-spread pricing expression derived in the paper yields the reinsurance price estimates. When we compare these prices to estimates provided by the MMC we observe that MMC prices are consistent with our estimates in that they embed tilting on the part of reinsurer. Similar observations are made when we assess aggregate coverage of loss risk by the FDIC. Our estimates of the average premium of 22.4 cents on $100 deposits reflects tilting the statistical distribution by a coefficient that is observable in the options market.

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# 7 Appendix

## 7.1 Proof of Proposition 1: Derivation of the Weibull call option model

Note that for strike $X$ we can express the payoff to a call option written on the loss level $L$ as follows:

$$C(X) = \int_X^\infty (L - X) f(L) dL, \tag{31}$$

where the Weibull probability density function is given by

$$f(L) = \exp\left(-\left(\frac{L}{c}\right)^a\right)\frac{aL^{a-1}}{c^a}. \tag{32}$$

Hence, equation (31) can be written as,

$$C(X) = \int_X^\infty L\exp\left(-\left(\frac{L}{c}\right)^a\right)\frac{aL^{a-1}}{c^a}dL - X\exp\left(-\left(\frac{X}{c}\right)^a\right). \tag{33}$$

The first term  is simplified as follows:

$$\int_X^\infty L\exp\left(-\left(\frac{L}{c}\right)^a\right)\frac{aL^{a-1}}{c^a}dL = \frac{a}{c^a}\int_X^\infty L^a\exp\left(-\left(\frac{L}{c}\right)^a\right)dL \tag{34}$$

Letting $u = \left(\frac{L}{c}\right)^a$, $y = cu^{\frac{1}{a}}$, and $dy = \frac{c}{a}u^{\frac{1}{a}-1}$, we have

$$= \frac{a}{c^a}\int_{\left(\frac{X}{c}\right)^a}^\infty c^a u\left(\exp\left(-u\right)\right)\frac{c}{a}u^{\frac{1}{a}-1}du, \tag{35}$$

$$= c\int_{\left(\frac{X}{c}\right)^a}^\infty u^{\frac{1}{a}}\left(\exp\left(-u\right)\right)du \tag{36}$$

$$= c\int_0^\infty u^{\frac{1}{a}}\left(\exp\left(-u\right)\right)du - c\int_0^{\left(\frac{X}{c}\right)^a} u^{\frac{1}{a}}\left(\exp\left(-u\right)\right)du \tag{37}$$

$$= c\Gamma\left(1+\frac{1}{a}\right) - c\Gamma\left(1+\frac{1}{a}\right)\frac{\int_0^{\left(\frac{X}{c}\right)^a} u^{\frac{1}{a}}\left(\exp\left(-u\right)\right)du}{\Gamma\left(1+\frac{1}{a}\right)} \tag{38}$$

Noting that,

$$gammainc(w,\gamma) = \frac{\int_0^w u^{\gamma-1}\left(\exp\left(-u\right)\right)du}{\Gamma(\gamma)}. \tag{39}$$

and substituting equation (38) in equation (33) and discounting it at the risk-free rate of $r$, we have

$$C = e^{-r} \square\, c\Gamma\left(1+\frac{1}{a}\right)\left(1 - gammainc\left(\left(\frac{x}{c}\right)^{a}, 1+\frac{1}{a}\right)\right] \tag{40}$$

$$- X\exp\left(-\left(\frac{X}{c}\right)^{a}\right)\Bigg)\Bigg]$$

## 7.2 A utility based derivation of the relation between $q(L)$ and $p(L)$.

The expected utility of an agent absorbing the loss payment $L$ is given by

$$u = E\left[U(W-L)\right] \tag{41}$$

$$= \int_{W,L} U(W-L)f(W)\lambda(W)g(L|W)dWdL$$
$$....... + \int_{W} U(W)f(W)\left(1-\lambda(W)\right)dW. \tag{42}$$

The agent's utility of the end of period wealth, $W$, with marginal distribution, $f(W)$, in the absence of a loss, $L$, is $U(W)$. This state has a probability of $(1-\lambda(W))$ where, $\lambda(W) = \int_0^\infty p(W,L)dL$ is the probability of a loss given the end of period no loss wealth of $W$.Here, $p(W,L)$ is the joint density for a loss level $L$. Hence, with $\lambda(W)$ probability, the agent is exposed to losses and his utility is $U(W-L)$.In this case, the conditional density of loss is given by $g(L|W) = \frac{p(W,L)}{\lambda(W)}$.

Now suppose that the loss level is independent of the end of period no loss wealth level and that

$$g(L|W) = p(L) \tag{43}$$

the unconditional density of loss given the existence of a loss. Also suppose that $\lambda(W) = \lambda$ a constant. We may then write

$$u = \int_{W,L} U(W-L)f(W)\lambda p(L)dWdL + \int_{W} (1-\lambda)\,U(W)f(W)dW. \tag{44}$$

Suppose now that the agent is offered a contingent claim paying $c(L)$ at the end of the period. If the agent were to take a position of $t$ units in this claim at the fair forward price of $a$ dollars then the expected utility of the agent can be expressed as:

$$\begin{aligned} V(t) &= \int_{W,L} U(W - L + tc(L) - ta) f(W) \lambda p(L) dL dW \\ & \quad .... + \int_{W} (1-\lambda) U(W - ta) f(W) dW \end{aligned} \tag{45}$$

Because the claim is fairly priced, we have that $V'(0) = 0$. Evaluating $V'(t)$ we get

$$V'(t) = \int_{W,L} U'(W - L + tc(L) - ta) f(W) \lambda p(L)(c(L) - a) dL dW \tag{46}$$

$$-a \int_{W} (1-\lambda) U'(W - ta) f(W) dW \tag{47}$$

Equating $V'(0)$ to 0 we get

$$\begin{aligned} & \int_{W,L} U'(W-L) f(W) \lambda p(L) c(L) dL dW \\ = \; & a \int_{W,L} U'(W-L) f(W) \lambda p(L) dL dW \\ & + a \int_{W} (1-\lambda) U'(W) f(W) dW \end{aligned} \tag{48}$$

It follows that the fair forward price, $a$, of the contingent claim is:

$$a = \frac{\int_{W,L} U'(W-L) f(W) \lambda p(L) c(L) dL dW}{\int_{W,L} U'(W-L) f(W) \lambda p(L) dL dW} \frac{1}{1+b} \tag{49}$$

where,

$$b = \frac{\int_{W} (1-\lambda) U'(W) f(W) dW}{\int_{W,L} U'(W-L) f(W) \lambda p(L) dL dW} \tag{50}$$

Define

$$q(L) = \frac{\int_{W} U'(W-L) f(W) \lambda p(L) dW}{\int_{W,L} U'(W-L) f(W) \lambda p(L) dL dW} \tag{51}$$

then we can write:

$$a = \int_{L} q(L) c(L) \frac{1}{1+b} dL \tag{52}$$

Equation (52) shows that, $q(L)$ is the risk neutral density for a loss level of $L$, given the existence of a loss while $(1+b)^{-1}$ is the risk neutral probability of a loss. Hence, we can establish the relation between statistical, $p(L)$, and.risk neutral, $q(L)$, probability distributions, assuming a specific utility function. For the case of an exponential marginal utility or the case of constant absolute risk aversion, $\alpha$,

$$U'(W) = \exp(-\alpha W) \tag{53}$$

equation (51) is written as

$$q(L) = \frac{\int_W \exp -\alpha(W-L)f(W)\lambda p(L)dW}{\int_{W,L} \exp -\alpha(W-L)f(W)\lambda p(L)dLdW}$$

simplifying we obtain:

$$q(L) = \frac{e^{\alpha L}p(L)}{\int_0^\infty e^{\alpha L}p(L)dL} \tag{54}$$

Note that if $U'$ is constant and utility is linear then $b = (1-\lambda)/\lambda$ and $(1+b)^{-1} = \lambda$ , which is the statistical probability of a loss. More generally we expect $U'(W-L) > U'(W)$ so $b$ should be less than $(1-\lambda)/\lambda$ and $(1+b)^{-1}$ is greater than $\lambda$. Hence the presence of risk aversion raises the risk neutral loss probability over its statistical counterpart.

For the aggregate system we suppose that $\lambda = 1$ and there is some loss each year and hence the equation for pricing loss contingent claims in the spot market is

$$e^{-r}a = e^{-r}\int_0^\infty q(L)c(L)dL$$

where risk neutralization occurs in accordance with equation (54).

**TABLE 1: Risk-neutral Weibull parameter estimates on extreme 2-month BKX call options and implied tilting coefficients in the BKX tail distribution.**

| Date | Maturity | curn | aurn | Tilt Coefficient |
|---|---|---|---|---|
| Oct. 2000 | .1804 | 0.0821 | 1.1453 | 0.1348 |
| Nov. 2000 | .1968 | 0.0176 | 0.4245 | 0.2533 |
| Dec. 2000 | .1779 | 0.0272 | 0.5128 | 0.2588 |
| Jan. 2001 | .1781 | 0.0540 | 0.6997 | 0.2415 |
| Feb. 2001 | .1779 | 0.0659 | 0.9728 | 0.1747 |
| Mar. 2001 | .1779 | 0.0914 | 1.0725 | 0.1841 |
| Apr. 2001 | .1753 | 0.0867 | 0.9985 | 0.2026 |
| May 2001 | .1945 | 0.0680 | 0.9862 | 0.1596 |
| Jun. 2001 | .1753 | 0.0524 | 1.0211 | 0.1135 |
| Jul. 2001 | .1945 | 0.0734 | 1.1855 | 0.0732 |
| Aug. 2001 | .1917 | 0.0637 | 1.1206 | 0.1152 |
| Sep. 2001 | .1563 | 0.0913 | 1.1556 | 0.1763 |
| Mean | .1814 | 0.0645 | 0.9413 | 0.1739 |

**Table 2. Tilt coefficients for a sample of bank holding companies**

Tilt coefficients are estimated for the September 1998- December 2002 period. Estimated weeks represent the number of weeks when at least three options exist for estimating the tilt coefficients. Average credit spread are calculated over weekly spreads between January 2000 and May 2001

| | Avg. Tilt | Median Tilt | Estimated Weeks | Avg. Number of Options | Average Credit Spread (%) |
|---|---|---|---|---|---|
| Bank of America Corp. | 0.278 | 0.281 | 102 | 14 | 1.746 |
| BB&T Corp. | 0.197 | 0.194 | 75 | 8 | 1.222 |
| Bank of New York | 0.198 | 0.187 | 87 | 12 | 1.529 |
| Citigroup | 0.141 | 0.130 | 98 | 15 | 1.433 |
| Comerica Inc. | 0.262 | 0.259 | 59 | 9 | 1.682 |
| Fleet Boston Financial | 0.232 | 0.217 | 85 | 9 | 1.625 |
| Fifth Third Bankcorp. | 0.232 | 0.236 | 61 | 8 | 1.237 |
| J.P.Morgan Chase | 0.179 | 0.176 | 98 | 15 | 1.593 |
| Key Corp. | 0.197 | 0.195 | 84 | 9 | 1.590 |
| National City Corp. | 0.298 | 0.291 | 71 | 7 | 1.845 |
| Bank One Corp. | 0.207 | 0.202 | 95 | 15 | 1.886 |
| PNC Financial Services | 0.251 | 0.244 | 77 | 11 | 1.346 |
| South Trust Corp. | 0.308 | 0.264 | 79 | 7 | 1.838 |
| Suntrust | 0.305 | 0.303 | 76 | 10 | 1.729 |
| State Street Corp. | 0.164 | 0.169 | 70 | 13 | 1.522 |
| Union Planters | 0.327 | 0.326 | 65 | 8 | 2.154 |
| US Bancorp. | 0.154 | 0.151 | 89 | 12 | 1.609 |
| Wachovia Corp. | 0.265 | 0.267 | 93 | 10 | 1.606 |

**TABLE 3: FDIC Annual Loss Levels**

Source: *Failed Bank Cost Analysis, 1986-2000*, Division of Finance, FDIC

| Year | Loss (in $Billions) | Number of Bank Failures |
|---|---|---|
| 1986 | 1.775 | 145 |
| 1987 | 2.023 | 203 |
| 1988 | 6.921 | 280 |
| 1989 | 6.199 | 207 |
| 1990 | 2.785 | 169 |
| 1991 | 6.148 | 127 |
| 1992 | 3.675 | 122 |
| 1993 | 0.646 | 41 |
| 1994 | 0.179 | 13 |
| 1995 | 0.085 | 6 |
| 1996 | 0.038 | 5 |
| 1997 | 0.005 | 1 |
| 1998 | 0.234 | 3 |
| 1999 | 0.841 | 7 |
| 2000 | 0.039 | 6 |

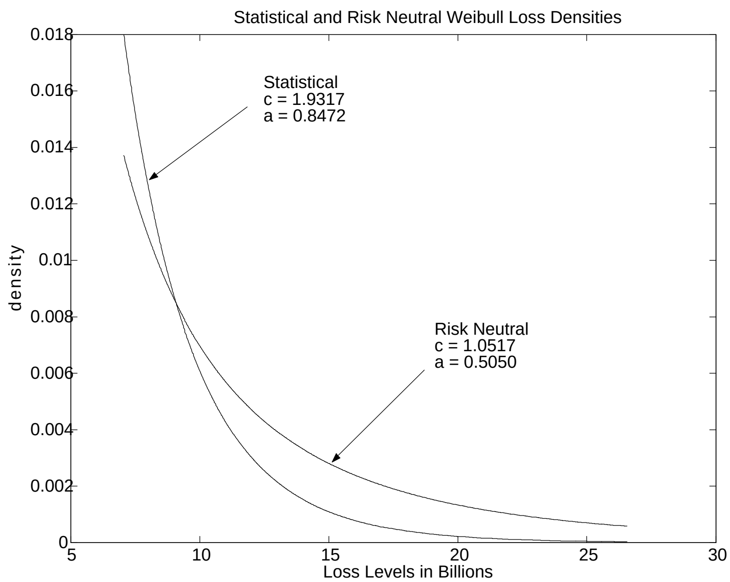


Figure 1: Weibull Statistical and Risk Neutral Distributions for Annual Fund Loss Levels. The risk neutral distribution is obtained by exponential tilting the statistical density using Bank Index option market tilt coefficient.