Financial Intermediation and Commitments to Optimal Investment Strategies

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Abstract

Reducing risks and enhancing liquidity are functions long emphasized in financial intermediation literature. However, financial intermediaries may also help savers commit to optimal investment strategies by limiting liquidity when investment opportunities warrant. We illustrate this sort of commitment function using a variant of the Diamond and Dybvig (1983) model in which the possibility that savers may need to liquidate their investments conflicts with long-run investment prospects. The assets considered here are completely tradable. Yet savers can do better if they delegate their investment decisions to an intermediary as a piggy bank of sorts that sets payments to optimize long-run returns, even though claimants know that in some states they may incur penalties. Although this piggy-bank function may not characterize any particular type of real-world intermediary literally, in the abstract the function may apply to a number of them. By preventing savers from reneging on contracts in the short run, our intermediary allows individuals to have better long-run outcomes. Thus, the function we emphasize may apply more particularly to intermediation that is associated with long-term savings goals such as saving for retirement.

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1. Introduction

At least three basic functions have been identified with financial intermediaries, including risk reduction, liquidity provision, and information production. This paper illustrates a fourth role, one in which intermediaries enable savers to commit to optimal long-run savings strategies by limiting withdrawals at inopportune times. We would argue that this commitment has not been emphasized in the literature—no doubt because credit risk diversification, liquidity provision, and information production are significantly more important functions of depository institutions (i.e., banks). Although banks may perform this fourth role to some extent, we view it as more broadly associated with intermediation earmarked for long-term saving objectives—such as retirement. Thus, our model is more illustrative of intermediation through pension and mutual funds or insurance contracts. Indeed, our intermediary can even be thought of as a sort of privatized social security fund, optimizing intergenerational (as opposed to intrapersonal) transfers of wealth. With regard to savers' long-term investment goals, we would argue that this role is relevant and interesting.

We illustrate this commitment function using a variant of the liquidity-shock model used by Bryant (1980) and Diamond and Dybvig (1983). In the standard versions of the model, the random demand for liquidity forces savers to trade off asset returns and liquidity; the long-term asset pays more, but the short-term asset pays sooner. Intermediaries emerge to provide a form of liquidity insurance to savers that want to smooth intertemporal consumption. They offer time-contingent deposit contracts that effectively shift some long-term asset returns to the unlucky savers that must liquidate investments so as to consume early.[1]

[1] Although this model was used initially to explore the reasons and effects for bank runs, it is now used more widely as a model of intermediation. See also Bernanke and Gertler (1987), Wallace (1990), and Hellwig (1994).

Our version of the model is deliberately altered to highlight the value of intermediaries as a means of committing to optimal investment strategies. In our model, savers do face unobservable liquidity shocks; they know there is some probability that they will need to liquidate their holdings early. But risk sharing in the sense of consumption smoothing is not as important for our savers; they are less risk averse regarding investment returns than the "depositors" considered in models emphasizing liquidity provision. Since our savers can costlessly invest in and trade short-term and long-term assets (no information production is required), they can fund their own portfolios and simply trade assets if they need to liquidate early. However, herein lies a second key difference in our variant of the model. Our short-term assets can be reinvested at returns that are random when portfolio decisions are made. And although expected returns are higher on long-term "illiquid" projects, short-term project returns exceed those on long-term projects in some states.[2]

The commitment problem and piggy-bank solution to long-term investment strategies reflect how uncertainty about future returns can adversely interplay with imperfect information about savers' true liquidity needs. Ex ante, before actual returns and liquidity needs are realized, savers will optimally plan to reinvest some of their liquid assets in the high-return state. But, ex post, the savers needing to liquidate, will renege on this strategy—consuming their short-term assets instead of rolling them over. Left to their own devices, savers violate the standard intertemporal consumption condition, $U'(c_1) = rU'(c_2)$, by consuming too much, too soon when the interest rate r turns out high. The inability to credibly commit to the optimal saving plan distorts savers' allocation of wealth across investment opportunities and lowers the return to savings and savers' welfare. Savers can overcome this commitment problem, or at least reduce

[2] While this assumption is key for our result, it seems more plausible than the constant returns often assumed.

it, by ‘‘locking up’’ their funds in a piggy bank of sorts. This piggy-bank intermediary limits withdrawals when short-term yields turn out to be high, increasing reinvestment when it is most profitable. Ex post, savers with liquidity needs lose out in this case, but ex ante, savers are better off limiting access to their funds and delegating investment decisions to a less myopic intermediary: thus, they do.

The commitment function in our model is very different from the other roles identified with intermediaries—particularly banks. Our intermediary is definitely not providing risk sharing in the sense of consumption smoothing. In fact, yields from this piggy bank are more volatile; and hence its benefit is greater, the more willing savers are to take on greater risk for greater return. Nor is our intermediary producing information about potential investments or diversifying away idiosyncratic project risks; the assets considered here are effectively securities that savers can hold directly and trade freely as liquidity needs and interest rates change. Nor, finally, is our intermediary providing liquidity; the problem here is really too much liquidity in some sense, and the solution is to lock up funds in an intermediary that limits liquidity in some states.

Of course, not all motives for saving require the sort of discipline provided by our piggy-bank intermediary. However, for certain types of investments at certain times, the long-run benefits of investing may be at odds with short-term incentives to withdraw funds. Earmarking funds to an insurance annuity or pension plan may provide a commitment value to some savers quite apart from, or in addition to, the other benefits associated with such intermediated contracts. Other real-world intermediaries may also help create incentives to stick to optimal investment strategies, quite apart from or incidental to their more familiar roles. Even banks are now issuing callable CDs having withdrawal penalties that are effectively contingent on interest rates—much like our conceptual intermediary[3].

Although the commitment function considered here might seem new, the notion of self-control or commitment problems among savers goes way back in the literature. Thaler and Shefrin (1981) model the commitment problem for two-sided savers—the far-sighted planner struggling against the short-sighted consumer. They trace this idea back to Adam Smith (1759). In his classic on investment, Strotz (1955) observed that investment plans might not be implementable if savers had discount rates that declined over time. Laibson's (1997) hyperbolic discounting is a modern variant of that idea, modeled as a game between selves over time. Barro (1997) solves a neoclassical growth model with hyperbolic discounting and works out the implications for savings, capital accumulation, and growth. He notes in passing that these sorts of commitment problems seem to have important institutional implications as well, which are basically the topic of this paper.

The next section describes the savings environment and optimal allocation; we characterize the solution for general preferences and illustrate with linear (i.e. risk-neutral) preferences. Section three contrasts that optimal allocation with the allocation achieved when savers invest directly and trade assets among themselves; we highlight the divergence from the optimal allocation and show how the resulting portfolio distortions (between short- and long-term assets) *decrease* with the degree of risk aversion. Section four shows how a financial intermediary can improve savers' welfare, even if the intermediary is not able to observe savers' true liquidity needs. We contrast the intermediary contract with a more complicated option-like

[3] Issuers can call the CD over a specified interval, either by paying off the CD holder or by rolling it over at the new market rate. All else equal, banks will exercise the call when rates have fallen and will pass when rates have risen, so the reinvestment decision by banks and spending decisions by savers will resemble those of our model savers to some extent.

arrangement in which savers hold long-term assets directly and buy options to sell these assets contingent on their liquidity needs. We conclude in section five. Proofs are in the appendix.

2. The Savings Environment

The savings and investment environment in our model is simple: in period zero, savers invest their endowment in short-term assets and long-term assets to support consumption in periods one and two. Like others, we assume savers are uncertain about their investment horizons. The main wrinkle in our setup is that long-term investors may want to roll over their short-term assets.

We model uncertainty about investment horizons in the same way that Bryant (1980) and others do. In period one, a fraction ρ of savers will realize a privately observed liquidity shock that forces them to consume immediately, while the remaining $1-\rho$ will wait until period two to consume. If consumption in period *t* is c_t, utility in period zero is

$$U(c_t) = \begin{cases} U(c_1) & \text{with probability } \rho \\ U(c_2) & \text{with probability } (1\text{-}\rho), \end{cases}$$

where $U'(.) \leq 0$ and $U''(.) \leq 0$. The liquidity shocks could be thought of as an irresistible impulse to consume, or as a drastic event, like illness or death, that forces savers to consume early. They provide a key friction in our setting (as in other models) as they are a tractable means of modeling withdrawal decisions that cannot be reversed simply by the adjustment of a market-clearing price. Allowing late consumers to substitute between period-one and period-two consumption would not change our basic result, but as in the rest of the literature using these types of liquidity shocks, the assumption that early consumers cannot postpone consumption until period two is important. As Hellwig (1994) states, ‘‘These [consumption] needs are inexorable ...there is no question of substitution between dates 1 and 2.’’

Consumption is supported by the returns from short-term and long-term investments made in period zero. Short-term assets yield one unit of output in period one per unit invested in period zero, and this output can be consumed or reinvested until period two. Long-term assets yield nothing in period one and one unit of output in period two (per unit invested in period zero).[4] Here, long-term assets are illiquid only in the physical sense that they yield no output in the short term; individuals can trade long-term assets for short-term assets, so they are liquid in the financial sense. This contrasts with models that assume long-term projects can be physically liquidated, at a loss, but are not tradable.

A key feature of our setup, however, is that in some states, long-term investors will want to roll over their short-term assets. We model this by assuming that the period-two return on the short-term asset is random:

$$r_i = \begin{cases} r_l < 1 & \text{with probability } \pi \\ r_h & \text{with probability } 1-\pi. \end{cases}$$

Savers learn this return in period one, at the same time they learn their investment horizon. In terms of expected returns, long-term assets dominate short-term assets,

$$\pi r_l + (1-\pi) r_h < 1.$$

Nonetheless, savers need to hold some short-term assets in case they must consume early. Whereas in much of the literature this is the only reason that savers hold short-term assets, in our setup savers will want to reinvest their short-term assets when their return is high. The option to

[4] The assumption of unit returns is not essential; it just keeps the notation cleaner.

reinvest is valuable only if the short-term asset return dominates the long-term asset return in the high-return state:

$$r_h > 1. \tag{1}$$

Assumption (1) is a fundamental difference in our setup; others generally assume the opposite inequality (or equivalently, that $\pi = 1$). Here we merely allow some probability that short-term assets can pay more than long-term assets if short-term rates rise sufficiently.

Privately observed liquidity shocks in tandem with the uncertainty about future returns on short-term, reinvestable projects generate the "commitment problem" that our financial intermediary helps to mitigate. We take asset returns as primitive factors determined outside the model. Investment and trading among agents, however, determine the relative price of the two types of assets and the equilibrium consumption allocations. We present three general equilibrium solutions: first we consider the planner's solution, then the allocations yielded by direct investment and trading in decentralized markets, and finally the intermediated solution.

The timeline below illustrates the sequence of investment, project returns, and consumption realizations.

Period

0	1	2
-invest in short-term and long-term projects	-short-term projects yield 1 per unit; long-term projects yield zero output	-long-term project pays 1 per unit invested in period 0
-return on short-term asset in period one is uncertain	-learn period 2 yield on short-term projects reinvested in period 1 (r_h >1>r_l)	-short-term projects yield r_j per unit reinvested in period one time 1
-consumption needs uncertain	-learn consumption needs: -early consumers swap long-term projects for output from short-term projects of late consumers and consume this output	-late consumers "eat" returns from long-term projects and reinvested short-term projects
	-late consumers reinvest any remaining short-term projects	

The First Best

In period zero, the planner collects each saver's unit endowment and invests s_o per saver in short-term assets and $1-s_o$ per saver in long-term assets. In period one, when the short-term rate r_i is realized, the planner can reinvest some fraction s_i in short-term assets until period two. Reinvestment will be state-contingent; hence the subscript. The planner also chooses consumption in each period, and consumption will be state-dependent as well. The period-one consumption constraint in state *i* is

$$c_{1i} = (s_o - s_i)/\rho.$$

Early consumers get the output from short-term investments less the amount that is reinvested. (Consumption is scaled up by $1/\rho$ because the planner invests s_o, but only ρ consume early.) Late consumers get the returns from long-term assets and from any short-term assets rolled over from period one; hence, period-two consumption in state *i* is

$$c_{2i} = (1 - s_o + r_i s_i)/(1-\rho).$$

The only other constraint facing the planner is that reinvestment in both states must be nonnegative:

$$s_i \geq 0.$$

Subject to these three constraints, the planner chooses $\{s_o, s_i, c_{1i}, c_{2i}\}$ to maximize the representative saver's expected utility in period zero:

$$EU(c_{ti}) = \pi[\rho U(c_{1l}) + (1-\rho)U(c_{2l})] + (1-\pi)[\rho U(c_{1h}) + (1-\rho)U(c_{2h})].$$

Except for the state-contingent aspect, this is a straightforward programming problem. The first-order conditions can be combined as follows:

$$s_i \lambda_i = 0, \tag{2}$$

$$U'(c_{1l}) - r_l U'(c_{2l}) = \lambda_l / \pi, \tag{3}$$

$$U'(c_{1h}) - r_h U'(c_{2h}) = \lambda_h / (1-\pi), \tag{4}$$

$$(1-\pi)[U'(c_{1h}) - U'(c_{2h})] = \pi[U'(c_{2l}) - U'(c_{1l})]. \tag{5}$$

where λ_i is the multiplier on the constraint that $s_i \geq 0$. Conditions (3) and (4) are the combined first-order conditions for c_{ti} and s_i, while (5) is the first-order condition for s_o.

Naturally enough, the planner's reinvestment strategy for the short-term asset is contingent on the rate realized in period one: if the rate is low, optimal reinvestment is zero; if high, reinvestment is positive (see appendix for proof).[5] Since $s_h > 0$ implies $\lambda_h = 0$, condition (4) becomes

$$U'(c_{1h}) / U'(c_{2h}) = r_h \tag{6}$$

in the high interest-rate state. This standard intertemporal efficiency condition has the planner transfer goods from period one to period two (by rolling over short-term assets) until the marginal (social) rate of substitution between the early and late consumption equals the marginal rate of transformation. Since $r_h > 1$, $c_{1h} < c_{2h}$, late consumers benefit more when the return on the short-term asset is high. On the other hand, $s_l = 0$ implies $\lambda_l > 0$, hence $c_{1l} > c_{2l}$; early consumers are compensated for their sacrifice in the high interest-rate state by more consumption in the low-rate state. Below, we show that when savers invest on their own, without benefit of a planner or an intermediary, they may violate (6); c_{1h} is too high, c_{2h} is too low, and reinvestment is inadequate when short-term project returns are high.

Substituting (6) into (5) yields

[5] Reinvestment in the low-return state is at least conceivable. If $r_h - 1$ were large enough relative to $1 - r_l$, the planner might invest only in the short-term asset in order to maximize reinvestment in the high-return state. With zero long-term investment, the planner must also reinvest in the low state to support late consumption. That strategy is suboptimal, however, because the short-term asset has a lower *expected* return. Likewise, zero reinvestment in the high-return state is also conceivable. Carrying over short-term assets means the planner could have invested more in the long term at time zero, which has the higher expected return. Nevertheless, the possibility of the high short-term return, and the saver's willingness to reallocate consumption, lead him to reinvest.

$$\pi[U'(c_{2l}) - U'(c_{1l})] = (1-\pi)[r_h - 1]U'(c_{2h}). \tag{7}$$

The left-hand side is the net marginal opportunity cost of investing in short-term assets when their return is low. In this state, the planner does not reinvest; he gives the unit return in period one to early consumers, which raises their utility by $U'(c_{1l})$. Had the planner invested in the long-term asset instead, he would have paid the unit return to the late consumers, which would have raised their utility by $U'(c_{2l})$. Since $c_{2l} < c_{1l}$, the opportunity cost of investing in short-term assets on the downside (when their return is low) will be positive.[6] The right side of (7) is the net marginal *benefit* of investing in short-term assets when their return turns out to be high. In this state, the planner reinvests in short-term assets and earns r_h. Had he invested more in the long-term asset in period zero, he would have earned only 1, so an extra unit of short-term investment raises late consumers' utility by $[r_h - 1]U'(c_{2h})$. The optimal portfolio equates the marginal cost of short-term investments on the downside with the marginal benefit on the upside—when their return is high.

The optimal portfolio and consumption allocations are determined in the following manner. Since $s_l = 0$ and $s_h > 0$, consumption in the low short-term rate state depends only on s_o, whereas consumption in the high-rate state depends on reinvestment as well:

$$c_{1l} = s_o / \rho,$$

$$c_{2l} = (1 - s_o)/(1-\rho),$$

$$c_{1h} = (s_o - s_h)/\rho,$$

$$c_{2h} = (1 - s_o + r_h s_h)/(1-\rho).$$

[6] The low short-term rate, r_l, does not enter the equation because the planner never invests at that rate; he pays out all short-term asset returns in that state to early consumers.

Substituting these equations into (6) and (7) determines the first-best amount of short-term investment and the optimal amount of reinvestment when short-term asset returns are high. This portfolio and reinvestment strategy then determines the first-best consumption allocation.

Risk-Neutral Savers: An Example

As we show below, the role for an intermediary depends on how the consumption allocation achieved by direct investment and trading compares with the first best. As we also demonstrate, the lower the degree of investor risk aversion, the worse the savers fare by direct investment. Hence, it is useful to consider the first-best allocation when savers are risk neutral. Suppose $U(c) = c$ for both early and late consumers. As usual with linear preferences, a corner solution is optimal: $s_0 = s_h = 1$. The planner invests only in short-term assets and reinvests the entire portfolio $(c_{1h} = 0)$ if the return turns out to be high; if the return is low, he pays out everything to early consumers

Although the corner solution is not surprising here, the particular corner is surprising; we thought that the planner might hold only long-term assets, since their return exceeds the expected return on short-term assets. With all long-term assets, however, savers get only $EU = 1,$ whereas the all-short-term portfolio yields $EU = \pi + (1-\pi)r_h > 1.$ What is key here is that the possible low return on short-term assets is never realized because in that state the planner pays out everything to early consumers. Thus, the effective return on short-term projects in the low-rate state is simply early consumers' marginal utility ($U'(c) = 1$ in this case). Thus, the all short-term portfolio does better because the planner avoids low returns by paying out everything to early consumers while he fully exploits high returns by reinvesting everything in that state. Risk-neutral individuals are completely amenable to an arrangement that structures payments to maximize the portfolio's overall return.

3. Direct Investment and Trading

There is nothing special about the assets here that would prevent savers from directly investing and trading on their own. But savers cannot generally replicate the planner's solution simply by funding their own portfolios in period zero and then trading assets after they observe short-term project returns and privately-observed liquidity shocks in period one. Below we characterize the decentralized equilibrium relative to the first best and show how the decentralized solution violates the planner's allocation.

A few preliminary observations will simplify the analysis. Since savers are identical ex ante, they all choose the same initial portfolio in period zero. In period one, early consumers will want to trade their long-term assets for short-term assets held by late consumers. The supply of long-term assets is perfectly inelastic because they yield no output until period two and therefore are worthless to early consumers. The demand for the long-term assets, however, will depend on the realized return to reinvesting short-term assets, as this return equals the opportunity cost of trading short-term assets to early consumers.

Let p_i denote the period-one price of a unit of long-term asset in terms of short-term assets in interest-rate state i. In period one, early consumers get the return on their short-term assets, plus the income from selling their long-term assets to late consumers:

$$c_{1i} = s_o + p_i(1 - s_0).$$

If each late consumer purchases l_i units of long-term assets in state i, period-one reinvestment in short-term assets is

$$s_i = s_o - p_i l_i.$$

Reinvestment cannot be negative:

$$s_i \geq 0.$$

This constraint ensures that each late consumer has enough short-term assets to cover his purchases of long-term assets. In state $i,$ late consumers get the unit return from their long-term assets (those they funded directly and those they purchased in period one) plus the period-two yield on reinvested short-term projects:

$$c_{2i} = 1 - s_o + l_i + r_i s_i$$

In period zero, savers choose $\{s_o, s_i, c_{ti}, l_i\}$ to maximize their expected utility, subject to the aforementioned constraints. The first-order conditions for c_{ti}, l_i, and s_i can be combined as follows:

$$s_i \lambda_i = 0 \tag{8}$$

$$(1/p_l - r_l)U'(c_{2l}) = \lambda_l / \pi(1-\rho) \tag{9}$$

$$(1/p_h - r_h \quad U \quad c_{2h}) = \lambda_h /(1-\pi)(1-\rho). \tag{10}$$

These conditions pin down the relationship between period-one reinvestment and the price of long-term assets in a given state. Since $s_i > \quad \Rightarrow \lambda_i = 0 \Rightarrow p_i = 1/r_i$; positive reinvestment in state i implies that the long-term asset price equals its discounted value in that state. (The price cannot be lower than $1/r_i$ or late consumers would sell short-term assets rather than reinvesting at r_i .) Conversely, $p_i < 1/r_i \Rightarrow \lambda_i > 0 \rightarrow s_i = 0 \Rightarrow p_i l_i = s_o$; hence, if the long-term asset's price is less than its discounted value, reinvestment is zero and long-term savers will trade all their short-term assets for long-term assets. In tandem these relationships imply that $p_i \leq 1/r_i$. (Obviously if $p_i > 1/r_i$, late consumers are better off not selling any short-term projects.)

Therefore,

The first-order condition for s takes two lines:

$$\begin{aligned}&(1-\pi)[\rho(1-p_h)U'(c_{1h})+(1-\rho)(r_h-1)U'(c_{2h})]+\lambda_h = \\ &\quad \pi[\rho(p_l-1)U'(c_{1l})+(1-\rho)(1-r_l)U'(c_{2l})]-\lambda_l.\end{aligned} \tag{11}$$

The left side is the expected net benefit of increasing s_o by another unit in the event that its return turns out high. Early consumers get the unit return less the opportunity cost of having one less long-term asset to sell in period one; but since $p_h < 1$, increasing s_o benefits early consumers. Late consumers also benefit on net, since they can reinvest and earn r_h>1. An additional benefit to late consumers is λ_h, the value of having more short-term assets to trade for long-term assets.

The right side of condition (11) is the benefit of increasing investment in the long-term asset by a unit in the event that short-term asset returns are low. Since $p_l > 1 > r_l$, investing more in long-term assets in period zero benefits both early consumers and late consumers in the low-return state. However, the $-\lambda_l$, reflects the cost to late consumers of not having more short-term projects to sell in period one.

In addition to these ex ante equilibrium conditions, the long-term asset market must also clear in both interest-rate states. In either state, the fraction ρ of the population will each supply $1-s_o$ long-term assets in period one; thus the aggregate supply of long-term assets is $\rho(1-s_o)$. In state *i*, the fraction $(1-\rho)$ of the population will each purchase l_i long-term assets; hence

[7] They would expect to buy long-term assets on the cheap in period one but would find that none was available, in which case the price would be unbounded, which contradicts $p_l < 1$.

aggregate demand for long-term assets in period one equals $(1-\rho)l_i$. Aggregate supply equals aggregate demand when

$$\rho(1-s_o) = (1-\rho)l_i. \tag{12}$$

To determine whether the spot market equilibrium equals the first best, we conjecture an equilibrium like the first best and then check if the resulting consumption allocations are indeed first best. A key feature of the first best is that the planner reinvests in short-term assets when their return is high. Accordingly, we conjecture $s_h > 0$, which implies

$$\rho_h = 1/r_h < 1. \tag{13}$$

At that price, consumption in the high-return state is

$$c_{1h} = s_o + (1-s_o)/r_h, \tag{14}$$

$$c_{2h} = 1 - s_o + r_h s_o. \tag{15}$$

Period-two consumption turns out to be independent of the amount of reinvestment when the long-term asset sells for its full discounted value; late consumers are indifferent between reinvesting or buying long-term assets at their market-clearing price. Relative consumption in the high-return state is

$$c_{1h} = c_{2h}/r_h. \tag{16}$$

For most preferences, this ratio will differ from the first-best ratio determined by (6), and the role for an intermediary depends on how these allocations differ.

Since reinvestment in the low-return state equals zero in the first best, we conjecture $s_l = 0$ here. If late consumers do not reinvest in the low state, they must trade all of their short-term assets for long-term assets:

$$p_l = \frac{s_o(1-\rho)}{\rho(1-s_o)} \tag{17}$$

The price of the long-term asset in the low-return state is increasing in the amount of period-zero short-term investment and the fraction of late consumers. More short-term assets means late consumers have more to ''spend'' on long-term assets and fewer to bid on. More late consumers means more buyers and fewer sellers. Since $p_l > 1,$ (17) implies that $s_o > \rho.$

The equilibrium price, portfolio mix, and other variables are determined as follows: Given that $s_l = 0,$ we know that $c_{1l} = s_o / \rho$ and $c_{2l} = (1-s_o)/(1-\rho).$ Given $s_h > 0,$ we know that $\lambda_h = 0$ and $p_h = 1/r_h.$ Substituting these values, along with (14), (15), and (9), into (11) produces an equation in s_o and p_l. This equation and (17) determine period-zero short-term investments and the period-one price of long-term assets when short-term returns are low. These values determine the consumption allocations in each state. Finally, given that $p_h = 1/r_h,$ s_o and (12) determine l_h and reinvestment in the high-return state: $s_h = s_o - p_h l_h.$

Risk-Neutral Savers

Recall that when $U(c) = c$, the planner invests only in the short-term asset even though it has a lower expected return. Clearly when investors are risk neutral, this first-best solution is not attainable through decentralized contracting (technically there is nothing to swap if $s_o = 1$). Even though individuals know that, ex ante, they would be better off promising to redistribute on the basis of whether it is optimal to consume early or to reinvest, they know that after the fact the ex post ''losers'' have no incentive to honestly reveal their type. If no transfers occur, then s_o = 1 yields $EU = \rho + (1-\rho)((1-\pi)r_h + \pi r_l) < 1.$

Savers will not choose to invest everything in long-term assets either. Suppose that they did and that in period one, the ρ early consumers simply handed over their long-term assets to late consumers (since long-term assets are worthless to early consumers). Each late consumer would receive $c_2 = 1 + (\rho/(1-\rho))$, and therefore $EU = (1-\rho)c_2 = 1$ for the all-long-term project portfolio.

Instead, risk-neutral savers, investing on their own, will reject both corners, choose a mixed portfolio in period zero, and then trade in period one. With linear utility, the first-order condition (11) is a function only of p_l.[8] At the equilibrium price, $\tilde{p}_l$, savers are indifferent to all values of s_o because, at this price, the downside opportunity cost of low short-term asset return exactly offsets the upside benefit of high returns. Substituting $\tilde{p}_l$ into (17) determines $\tilde{s}_o$, the portfolio consistent with that price. This equilibrium is unique because p_l is increasing in s_o. The equilibrium is also stable; if savers expected $p_l > \tilde{p}$, they would raise s_o and p_l would fall; if they expected $p_l < \tilde{p}$, they would lower s_o and p_l would rise. Although at $\tilde{p}$ savers are indifferent about s_o, their expected consumption given $\tilde{s}_o$, and $\tilde{p}$ is strictly higher than at any other combination. However, although this mixed portfolio dominates either corner solution, it yields lower expected utility than the planner's solution; for $0 < s_o < 1$, $EU = \pi + (1-\pi)[s_o(\rho + (1-\rho)r_h) + (1-s_o)((1-\rho) + \rho/r_h)] > 1$ is less than $EU = \pi + (1-\pi)r_h$, as yielded by the first best.

3.1 Risk Aversion and Investment Distortions

Here we characterize how direct investment and trading fail to generate the first-best allocation under more general preferences. Suppose

[8] We can replace p_h with $1/r_h$ since savers will reinvest in the high state, s_o.

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha} \tag{18}$$

where higher $\alpha \in [0,1)$ indicates increasing risk aversion. Equation (16) then implies

$$U'(c_{1h}) = r_h^{\alpha} U'(c_{2h}) < r_h U'(c_{2h}).$$

When savers invest directly in period zero, early consumption in the high-return state is higher than in the first best; thus there is too little reinvestment. The inequality implies that the planner could raise welfare by taking a unit from early consumers, reinvesting at r_h, and giving the proceeds to late consumers (but of course, ex post, early consumers will not voluntarily make this trade).

The lower the degree of savers' risk aversion, the greater the commitment problem and the inefficiency associated with direct investing and trading. The underpinning of the distortion is that the ratio c_{2h}/c_{1h} is independent of α when savers invest directly because relative consumption is pinned down by the relative returns on traded assets; the first-best ratio c_{2h}/c_{1h}, however, is a function of risk preferences (here, decreasing in α). Less-risk-averse individuals are more flexible about shifting consumption in order to exploit high returns, but their inability to commit ex post to the optimal savings plan leads them to underinvest in these projects. Savers' incentives are also distorted on the downside (when the short-term asset return is low), again because the market price they face differs from the shadow price facing the planner.

If, as in some models, late consumers could be limited in terms of what they could trade for long-term asset markets when reinvestment returns were high, p_h could be reduced and reinvestment increased. Although this limited participation would reduce the liquidity of the long-term asset, as in Diamond (1997), reduced liquidity in this case would ameliorate the commitment problem, as in Laibson (1997).

Upside-Downside Distortions

Savers underinvest in short-term assets because their upside benefit is too low and their downside cost (when their return is low) is too high. We illustrate these distortions for the risk preferences given by (18). Even for these standard preferences, the extent of nonlinearity in the first-order conditions precludes a closed-form solution for s_o. Instead, we reduce the first-order conditions to a single equation in s_o and compare the properties of the first best with those of the decentralized market solution. Using (6) to eliminate s_h from (7) produces the solution for the first best:

$$\pi f(s_o) = (1-\pi)(1-1/r_h)\, j(r_h)\, g(r_h s_o). \tag{19}$$

Let s_o^* denote the solution to (19). The π on the left side and the $(1-\pi)$ on the right side indicate that the planner determines s_o^* by weighing the downside of low returns with the upside of the high-return state. On the downside, the marginal opportunity cost of the short-term asset is

$$f(.) \equiv \left(\frac{1-\rho}{1-s_o}\right)^{\alpha} - \left(\frac{\rho}{s_o}\right)^{\alpha},$$

where f is positive (since $s_o > \rho$) and increasing in s_o. This expression is the net gain in utility from having invested in another long-term asset and giving the return to late consumers. The benefit when the short-term asset return is high depends on

$$g(.) \equiv 1/(1+(r_h-1)s_o)^{\alpha},$$

which equals the gain in utility from having another short-term asset to reinvest at r_h (and pay out to late consumers). Note that g is decreasing in s_o.

For the market solution, an equation comparable to (19) can be obtained by the substitution of the equilibrium long-term asset prices (12) into condition (11) (along with the substitutions described above):

$$\pi h(s_o) = (1-\pi)(1-1/r_h)k(r_h)g(r_h s_o). \tag{20}$$

The marginal cost of holding another short term asset when its return is low equals

$$h(.) \equiv \left[(1-s_o)\left(\frac{1-\rho}{1-s_o}\right)^{\alpha} + s_o\left(\frac{\rho}{s_o}\right)^{\alpha}\right]\left(\frac{s_o-\rho}{s_o(1-s_o)}\right).$$

Increasing s_o has two partially offsetting effects on $h(.)$. The term in the square brackets is the weighted average of late consumers' and early consumers' respective marginal utilities, where the weights are the respective portfolio shares held in long-term and short-term investments. Raising s_o lowers this term since this shifts more weight to early consumers, whose marginal utility declines with s_o. The term on the far right reflects the fact that increasing s_o increases the value of the long-term assets (since there are fewer of them), which directly increases the opportunity cost of funding another short-term asset. The net effect is positive: h(.) is increasing in s_o.

A comparison of benefits and costs indicates why savers underinvest in the short-term asset when they invest directly. The difference in the benefits of investing in short-terms project on the upside (when short-term project returns are high) can be found by comparison of

$$j(.) \equiv (\rho r_h + (1-\rho) r_h^{1/\alpha})^{\alpha} \quad \text{with}$$

$$k(.) \equiv \rho r_h^{\alpha} + (1-\rho) r_h.$$

Because x^{α} is concave and $\rho < 1, j > k$.[9] The upside benefit is lower with direct investment because late consumers pay too much for long-term assets and therefore reinvest too little. The difference in the downside costs, for a given $s_{o,}$, equals

$$h(s_o) - f(s_o) = (p_l^{\alpha} - 1)(\frac{\rho}{s_o})^{\alpha+1},$$

where p_l is the price of the long-term asset in the low interest-rate state. The shadow price of s_o to the planner equals one (he could have invested in long-term projects, each yielding one unit). But $p_l > 1$ in the market solution, so the marginal cost of holding short-term assets (that is, the value of having a long-term project) in the low-rate state is also higher under direct investment than for the planner. Both higher costs on the downside and lower benefits on the upside cause savers to underinvest in short-term assets.

The sources of the investment distortion are illustrated in Figure 1. Note that the distortions on each side are independent. Suppose savers have access to a better short-term asset with a higher upside return, r_h, but the same downside risk, π. The benefit curve shifts up with r_h, but the cost curve does not change. Suppose the upside on the new asset is high enough to compensate direct investors for their inability to fully exploit the high return. In this case, the upside benefit on the new asset to savers is coincident with the upside benefit on the old asset to the planner.[10] Savers would choose $s_o^{'}$ of the new short-term asset, less than s_o^{*}, because the cost on the downside would still be too high (relative to the planner's allocation). Again, note that if participation by long-term asset buyers were limited, as in Diamond (1997), the long-term asset price would be lower and the distortion would be reduced.

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[9] Let $x = r_h$ and $x' = r_h^{1/\alpha}$. Then $j = (\rho x + (1-\rho)x')^{\alpha} > \rho x^{\alpha} + (1-\rho)x'^{\alpha} = k$.

[10] If the upside on the new asset is r_{hh}, the benefit curves are coincident if $(1- \quad r_{hh}\ k\ r_{hh}\ g\ r_{hh} s_o \; = \; - \ /r_h) j(r_h) g(r_h s_o)$.

The difference between the optimal portfolio and savers' direct investments is decreasing in the degree of risk aversion (Figure 2). The only case in which the distortion is zero is when $U = \ln c$. In this case, (6) implies $c_{2h} = r_h c_{1h}$, the relative consumption yielded by direct investment and trading. We stress this to emphasize that our financial intermediaries are not providing insurance in the sense of consumption smoothing.

4. Financial Intermediaries

Delegating investment decisions to an intermediary can raise savers' expected utility and may allow them to realize the first-best level of welfare. In period zero, savers deposit their endowment with the intermediary, who offers them a two-period state-contingent contract. The contracting problem facing the intermediary is the same as the allocation problem solved by the planner, except that the intermediary cannot observe a given saver's realized liquidity needs in period one.[11] Privately-observed liquidity needs create an incentive problem: if those who withdraw early earn too much, long-term savers will pose as early consumers to withdraw early and reinvest on their own until period two. To prevent this, the contract offered by the intermediary must satisfy the following incentive constraint:

$$r_i c_{1i} \leq c_{2i}. \tag{21}$$

The intermediary's problem is a constrained version of the planner's problem. Thus, to test whether the intermediary can attain the first best, we need to test only if the additional constraint binds in either state in the first-best allocation. Savers' preferences will influence whether it does. If $U(c) = c^{1-\alpha}/(1-\alpha)$ and α<1, the incentive constraint does not bind when the returns to

[11] There are no fixed costs to intermediation, so there is free entry. Competition among intermediaries ensures that the survivors must offer contracts that maximize savers' expected utility at time zero.

reinvestment are high, since (6) implies $c_{2h} = r_h^{1/\alpha} c_{1h} > r_h c_{1h}$. Indeed, the optimal allocation is to penalize early consumers when the return to reinvesting is high, so late consumers certainly have no incentive to misrepresent their type in this state.

The incentive constraint may bind in the low-return state, however. The planner does not reinvest in that state, so the ratio c_{1l} / c_{2l} may be high enough to violate (21). If so, the intermediary must reduce short-term and increase long-term investment.[12] Substituting $c_{1l} = s_o / \rho$ and $c_{2l} = (1 - s_o)/(1-\rho)$ into (21) determines the intermediary's maximum incentive-compatible level of short-term investment:

$$\bar{s}_o = \frac{\rho}{\rho + (1-\rho) r_l}.$$

The upper limit on short-term investment falls as r_l rises; a higher downside increases the return to reinvestment, so the intermediary must reduce the amount available for late consumers to reinvest. Higher ρ increases the upper limit because short-term assets are distributed across more consumers in the low-return state.

Figure 1 shows that the intermediary achieves the first best if $\bar{s}_o$ falls to the right of s_o^*. If $\bar{s}_o > s_o^*$, the downside cost of the short-term asset exceeds the upside benefit, so the intermediary will not want to hold $\bar{s}_o$ in the first place. If $s_o^* > \bar{s}_o$, the intermediary cannot

[12] Reinvesting in the short-term asset will also relax the incentive constraint, but investing more in the long-term asset is the most efficient way to raise period-two consumption. Reinvestment is determined by (6) even if the incentive constraint binds.

achieve the first best, but he still dominates direct investment except when savers happen to choose $\bar{s}_o$ on their own.[13]

The intermediary described here serves a very different role from that considered by Diamond and Dybvig (1983). Their intermediary shifts the higher return from longer-term assets to early consumers so as to smooth consumption over time. Our intermediary does just the opposite; it allows savers to increase their portfolio returns by making early withdrawals contingent on their opportunity cost in terms of foregone investment returns.

4.1 Necessary Conditions for Intermediation

As with other models of this type, unobservable liquidity shocks are necessary for savers to need an intermediary (or a social planner). If liquidity shocks are observable, then savers can achieve the first best through direct trading as long as contracts can be enforced.[14] In addition, liquidity shocks must be such that they necessitate the complete liquidation of one's investments, irrespective of price; early consumers must get zero utility from postponing consumption. Suppose instead that savers get utility from consuming in both periods, but we define early consumers as people with relatively high marginal rates of substitution between early and late consumption, $U'(c_1)/U'(c_2)$. In that case, savers could still invest directly in the short-term asset. After they learned rates, both types would $U'(c_1)/U'(c_2)=r_2$. Consumption allocations

[13] Evaluating (19) at $\overline{s_o}$ produces a condition on the parameters that determines whether the intermediary achieves the first best: $s_o^* > \overline{s_o} \Leftrightarrow \dfrac{\pi(1/r_l^{\alpha}-1)}{(\rho r_h+(1-\rho)r_l)^{\alpha}} > (1-\pi)(1-1/r_h)\,j(r_h)$.

[14] If there is uncertainty only about returns but not about the timing of consumption needs, savers can do just as well by investing directly. Suppose savers know whether they will need to consume in period one or two. The solution is trivial; all early consumers invest only in the short-term asset whereas late consumers invest only in the long-term asset, since the latter has the highest expected return. In this case there is no role for an intermediary (or a planner).

would differ, but with the marginal rate of substitution for each type equal to the marginal rate of transformation ex post, there would be no gains to locking up short-term assets with an intermediary. Putting it differently, savers would make the same investments in period one that they would make in period zero, so there would be no commitment problem and no need for an intermediary.

4.2 Another Option

As in Boyd and Prescott (1986), the intermediary in our model is essentially a contract among a coalition of savers. The contract is not unique. Savers could invest directly in the long-term asset and buy an option to sell it the next period in case they needed to consume early. This would solve the essential purpose of the intermediary: locking up liquid assets—out of the reach of early consumers. This forward market does as well as the intermediary, but the arrangement is certainly more complicated.

The option market works as follows: In period zero, investors divide their wealth between short-term assets and long-term assets. They can also buy an option to sell long-term assets in period one for a price of q_i in state *i*. Late consumers, who do not exercise this option, will get a rebate of R_i in state *i*. In period zero, savers pay a dealer a fee of f for this option, and the dealer invests this fee in short-term assets. Fee revenues must be sufficient to buy up all the early consumers' long-term assets in period one at the stipulated price. Any excess revenue is reinvested in short-term assets, and the proceeds from reinvestment are rebated to late consumers along with the long-term assets purchased from early consumers.

By manipulating the budget constraints of the consumers and dealers, we can show that the equilibrium contract can improve upon the direct investment allocation.[15] The trick is to show that combined constraints on savers and the dealer reduce to the consumption constraints on the planners' problem. This option contract gives early consumers

$$c_{1l} = q_i(1 - s_o) + s_o.$$

The dealer must collect enough in fees to purchase the long-term assets from the ρ savers that will need to sell in period one: $f \geq \rho q_i l_o$. Let s_i denote any excess fees that are reinvested in short-term assets. The dealer's constraint in period one is

$$s_i \equiv f - \rho q_i l_o \geq 0.$$

Eliminating $q_i l_o$ from these two equations implies

$$\rho c_{1_i} = f - s_i + \rho s_o.$$

Consumption by late consumers is

$$c_{2i} = 1 - s_o + r_i s_o + R_i. \tag{22}$$

The dealer must earn enough on reinvestment and period-one purchases of long-term assets to pay the period-two rebate to the $(1 - \rho)$ savers that consume late:

$$(1 - \rho)R_i = r_i s_i + \rho(1 - s_o).$$

Eliminating R_i from these two equations implies

$$(1 - \rho)c_{2i} = 1 - s_o + r_i s_i + (1 - \rho)r_i s_o. \tag{23}$$

[15] Since the option dealer cannot observe the savers' type, he faces the same incentive constraint as the intermediary, (21). This condition ensures that late consumers will not exercise the option and then reinvest the proceeds in short-term assets. This constraint will be just as tight under the option market as under the intermediary. Since the option price will differ from the price that would prevail on the spot market, late consumers might also have an arbitrage opportunity; they could exercise the option to sell their long-term assets and then repurchase them from early consumers on the spot market. But the spot market will never open. If $q_i < p_i$, long-term consumers would not want to buy on the spot market. And if $q_i > p_i$, early consumers would not want to sell on the spot market.

Note that if $s_o = 0$ (that is, savers do not directly invest in any short-term assets), the consumption constraints (22) and (23) are the same as in the planner's problem. Let s^* denote the first-best level of investment in short-term assets. The solution to the option contract is the same, as long as $f = s^*$. However, like the intermediary, the dealer also faces an incentive constraint limiting high payouts to early consumers.

The crucial feature of this scenario is that savers do not hold short-term assets directly. This prevents early consumers from consuming too much when it is optimal to reinvest. Direct investment in the long-term assets poses no problems because savers do not have discretion regarding the timing of their consumption. Unlike the intermediary, who invests in both the short-term and long-term asset in period zero, the option dealer invests only in short-term assets. Still, the dealer equilibrium is intermediated in some sense, as savers give over a portion of their wealth to a third party, who invests it, and they trade no other assets amongst themselves.

5. Conclusion

Much of the intermediation literature focuses on mitigating the variability of possible outcomes—a standard view of what it means to enhance the liquidity of investments or provide insurance. Here we discuss a role that intermediaries can play in facilitating optimal investment strategies even (and especially) when investors are not particularly risk averse. Although our savers face liquidity shocks, they do not want the same sort of consumption smoothing that Diamond and Dybvig (1983) model in characterizing contracts more akin to traditional bank deposits. Nonetheless, although assets are completely tradable, savers can do better if they delegate their investment decisions to an intermediary as a sort of piggy bank that overrides savers' liquidity demands when investment opportunities warrant. Indeed, one might call this sort of commitment role "ensurance" in the sense that intermediaries can achieve a better investment outcome for savers (given their preferences) by ensuring that liquidators honestly reveal their type.

We do not aim to diminish the standard functions of intermediaries emphasized in the literature. Rather, here we use a relatively standard type of model to illustrate an additional role that an intermediary can play. This role involves funding optimal investment strategies that investors cannot achieve on their own because of the possibility that, ex post, some investors will regret making what was an optimal investment decision ex ante. The bottom line is that savers can achieve a better (expected) investment outcome by delegating their resources to the intermediary to implement the ex post payoffs.

Although this piggy bank function may not apply literally to any particular type of real-world intermediary, in the abstract it may apply to all of them. By restricting access to funds in the short run, intermediation may help individuals achieve better long-run investment returns—resisting incentives to withdraw at inopportune times. Thus this model builds on the more general idea that locking funds in an intermediary, whether a bank CD, a pension plan, or an insurance annuity, may help some savers stick to long-term plans that they would not commit to if they held funds directly. Obviously, these intermediaries attract funds for other reasons as well—namely, the higher or better-diversified returns that they offer. Our point is simply that the liquidity premium demanded by some savers may not be as large as one might expect since, for some portfolio choices, illiquidity may be a blessing in disguise.

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Appendix

This appendix states and proves four lemmas useful in simplifying the planner's first-order conditions for the optimal investment and consumption allocations.

Lemma 1 $s_l > 0 \Rightarrow s_h > 0$. Proof: assume $s_l > 0$; $s_h = 0$. Then $c_{1l} > c_{2l}$ *and* $c_{1h} < c_{2h}$. It follows from $c_{1l} > c_{2l}$ and $r_l < 1$ that $(1 - \rho) / \rho > (1 - s_0 + r_1 s_l) / (s_0 - s_l) > (1 - s_0) / s_0$ but $c_{1h} < c_{2h} \Rightarrow (1 - \rho) / \rho > (1 - s_0) / s_0$, a contradiction.

Lemma 2 $s_l > 0 \Rightarrow c_{2l} < c_{2h}$. Proof: Assume that $s_l > 0$ and $c_{2l} > c_{2h}$. Lemma 1 implies that $c_{1l} > c_{2l}$ and $c_{1h} < c_{2h}$. Thus, $c_{2l} > c_{2h}. \Rightarrow c_{1l} > c_{2l} > c_{2h} > c_{1h}$. But $c_{2l} > c_{2h} \Rightarrow r_l s_l > r_h s_h$ $\Rightarrow s_l > s_h$ (since $r_l < r_h$) $\Rightarrow c_{1l} < c_{1h}$, a contradication.

Lemma 3 $s_l = 0$. Proof: $s_l > 0 \Rightarrow$ $s_h > 0$ (by Lemma 1), hence $\lambda_l = \lambda_h = 0$. Conditions (3) - (5) $\Rightarrow (1 - \pi)(r_h - 1) / \pi(1 - r_l) = U'(c_{2l}) / U'(c_{2h}) > 1$ (since $c_{2l} < c_{2h}$ from lemma 2), but $\pi r_h + (1 - \pi) r_l < 1 \Rightarrow (1 - \pi)(r_h - 1) / \pi(1 - r_l) < 1$, a contradiction.

Lemma 4 $s_h > 0$. Proof: Suppose $s_h = s_l = 0 \Rightarrow c_{1l} = c_{1h} = c_1$ and $c_{2l} = c_{1h} \equiv c_2$. Condition (5) $\Rightarrow U'(c_1) = U'(c_2)$, but condition (4) $\Rightarrow U'(c_{1h}) > U'(c_{2h})$, a contradiction.

Figure 1

The Optimal Portfolio and the Distortions on the Upside and Downside

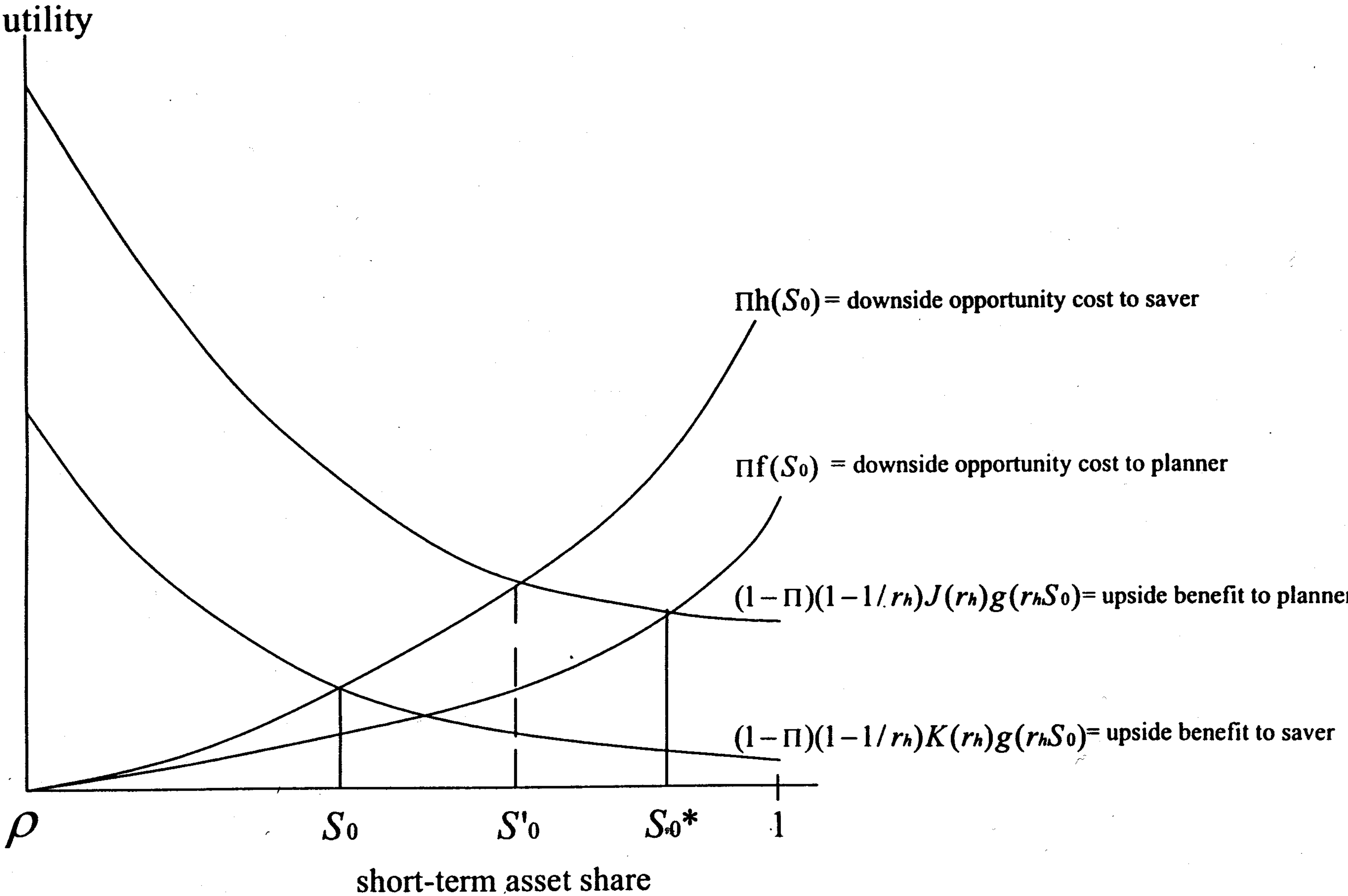

Figure 2
Risk Aversion and the Portfolio Distortion