FDIC Center for Financial Research Working Paper

No. 2007-02

Designing Countercyclicaland Risk Based Aggregate Deposit Insurance Premia*

November 2006

Federal Dposit Insurance Corporation • Center for Financial Research

November, 2006

Designing Countercyclical and Risk Based Aggregate Deposit Insurance Premia*

Abstract

This paper proposes an aggregate deposit insurance premium design that is risk-based in the sense that the premium structure ensures the deposit insurance system has a target of survival over the longer term. Such a premium system naturally exceeds the actuarily fair value and leads to a growth in the insurance fund over time. The proposed system builds in a swap in premia that reduces premia when fund size exceeds a threshold. In addition, we build in a swap contract that trades premia in good times for relief in bad times.

Robert A Jarrow, Dilip B. Madan and Haluk Unal

Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853,

Robert H. Smith School of Business, University of Maryland, College Park, MD 20742

and

Center For Financial Research, FDIC

raj15@cornell.edu, dmadan@rhsmith.umd.edu and hunal@rhsmith.umd.edu

*We greatly appreciate the comments of Robert DeYoung, Paul Kupiec, Art Murton and seminar participants at the FDIC.

1 Introduction

The Federal Deposit Insurance Reform Act of 2005 permits the FDIC to charge every bank a premium based on risk, provide initial assessment credits to banks that helped to build up the insurance funds, and require the FDIC to pay rebates if the ratio of insurance fund size to insured deposits (reserve ratio) exceeds certain thresholds. While theoretically very appealing, concerns have been raised about the procyclical adverse impact of risk based premia and capital requirements: regulations require banking organizations to pay higher premia in economic downturns, thereby aggravating the effects of economic recessions (Blinder and Wescott, 2001). Pennacchi (1999) provides evidence that during recessions banks reduce deposits to mitigate the effects of higher premiums, which in turn reduces bank credit.

With the exception of Pennacchi (2006) not much research exists that proposes methods to mitigate the effects of the cyclical movements in deposit insurance premiums.[1] Pennacchi shows that one way to minimize the adverse effects of the cyclical premiums is to set the fair insurance rates as a moving average of future losses. We add to this scant research and propose an aggregate premium policy that is countercyclical by design and is nonetheless founded on risk based principles.

The basic idea is to build into the premium system a swap contract that trades premia in good times for relief in bad times. In addition, the proposed design does not allow the deposit insurance fund size to grow excessively in a prolonged economic boom. This feature is engineered by also incorporating a premium reduction swap when the fund size exceeds a target level. The design therefore builds into the aggregate premium policy an elasticity for both downturns and fund growth, with premiums taking a percentage reduction in response to the depth of the downturn and the excess fund size.

The target fund size, aggregate premium level and rebate structures are all risk based by

[1] The countercyclical effects of capital regulation has attracted a wider reserach interest. Kashyap and Stein (2004), Allen and Saunders (2004), Gordy and Howells (2004), and White (2006) provide a review of these studies.

ensuring that the deposit insurance system has a high probability of survival over the longer term. We first establish a benchmark case where there is no premium rebates. In such a system we determine the target fund size and aggregate premium level that ensures a long-term fund survival. We then evaluate the effects of altering rebates and premium levels and fund-size targets in the neighborhood of the benchmark level on the probability of long term fund survival. We report the trade-offs that are implicit between these policies when we enforce neutrality of the long term survival probability.

We determine the long term survival probability by simulating the operation of the system through time. For the determination of these basic trade-offs we consider a time homogeneous formulation of the risks involved and we essentially perform all calculations in a steady state economy where deposit growth matches the spot rate of interest. The risks facing the system are the time series of losses on which the system has to payout and we build for this purpose a model for the aggregate loss distribution. The model entails three uncertain components. They are the number of loss events (bank failures), the asset size of the failing banks, and the loss rate given default.

The model employs the unconditional distributions of these events. We model the number of bank failures by a Poisson process with a constant arrival rate. For the distribution of asset sizes, we analyze the data on bank asset sizes for US banks over the years 2000-2003. We confirm that the Frechet distribution provides a statistically good fit for the data. With respect to loss rates, we analyze the data on loss rates experienced by the Federal Deposit Insurance Corporation over the period 1984-2002 on 1508 bank failures. We show that the Weibull model provides a statistically good fit to describe the historical loss experience.

We build these loss components into a simulation of the effects of alternative premium policies and evaluate the probability of long term fund survival resulting from the adoption of these risk based and countercyclical premium policies. The final result demonstrates the trade-offs between the various policy dimensions for a given target long term survival probability. The specific policy dimensions are the downturn rebate, the rebate for excessive growth of the fund, and the level of aggregate premiums and target fund size.

Results show that cyclicality in premiums can be diminished at a cost over time. For example, we show that, given our simulation assumptions, a loss and capital rebate system increases the aggregate premium about \$.9 billion on average over ten-years to mitigate the impact of the cyclicality over this period. This additional premium represents a 35% increase in \$2.6 billion premium for \$3.3 trillion aggregate deposits.

The outline of the paper is as follows. Section 2 presents the design of the countercylical and risk based premium system. In Section 3 we present the results on modeling the data on asset sizes and loss rates. In Section 4 we describe the simulation of the long term survival probability and establish the benchmarks for fund size and aggregate premiums. Section 5 constructs the trade-off table. Section 6 concludes.

2 The Aggregate Premium System

We assume that market for the bank failure losses is arbitrage free, and that there exists a risk neutral measure useful for valuation. The market for bank failure losses need not be complete. In an incomplete market, there is non-uniqueness of the risk neutral measure. A discussion of the appropriate choice of a risk neutral measures in an incomplete market is contained at the end of this section. For simplicity, we price the FDIC insurance premiums in aggregate, rather than pricing each individual's bank premium and summing across all banks. This is without loss of generality. To begin, we select one of the risk neutral measures (in a complete market, this is given) and we let $\widetilde{E}_t(\cdot)$ be its expectation operator, while *prob* denotes probabilities under the statistical measure.

For the analysis of the risks involved we adopt a discrete time model with annual periods denoted by $n = 0, \cdots, N$. Let L_n be the level of losses, in billions of dollars, paid out by the

Insurance fund in period n. The losses occur at the end of period. Let κ_n be the aggregate premium, in billions of dollars per year, paid at the beginning of the period. Let r_n be the spot rate of interest for period n (from the beginning to the end of the period). The spot rate for time n is known at time n, but future spot rates ($t > n$) are random variables when viewed from time n. Then, risk neutral valuation implies that the fair insurance premium is:

$$\kappa_n = \frac{\widetilde{E}_n(L_n)}{1 + r_n}. \tag{1}$$

If one believes that the FDIC insurance fund is large enough that bank failure risk is diversifiable, then it can be argued that one should set annual premiums at their actuarially fair level or equal to the statistically expected loss level, in which case the risk neutral measure should be replaced by the statistical measure in the previous expression. More generally, for the pricing of optionalities on L_n under the statistical measure one needs a strong form of diversifiability and we refer the reader to Atlan, Geman, Madan and Yor (2006) for further details.

The premium κ_n, as determined by the previous expression, has two properties that need to be emphasized. The first property is that the premium is determined independently of the size of the FDIC insurance capital fund. This follows from the fair pricing argument. And, the second property is that the premium is time varying, changing based on the fluctuations of both varying market conditions (as reflected in the information set and the risk premia reflected in the change of probability embedded in the risk neutral conditional expectation operator) and the spot rate of interest.

Let us address the first property. The FDIC can be thought of as a governmental "trustee" for the banking industry, whose duty is to manage/oversee a fund for insuring bank failures. The banking industry itself is responsible for funding the entire cost of the insurance and the expenses of the trustee. The trustee's duties are to set fair premiums, to collect the premiums, and to pay the losses due to bank failures. Because losses may exceed premiums in any period, the trustee is also required to collect an initial "maintenance account" (provided by the banking industry) at the start of the Insurance fund which will earn a fair return and from which excess losses will be paid. If the premiums exceed looses in any year, then the trustee should also rebate these excess premiums to the banking industry.

The Insurance fund size is set so that the fund's principal and the premiums collected should, with a certain confidence level over a prespecified period of time, not be depleted. Depletion implies fund insolvency. If depleted, then the "trustee" will have to go back to the banking industry to make up any shortfall and to re-establish the principal within the Insurance fund. Because insolvency of the fund would be a costly event, both politically and financially, the confidence level and the time horizon should be set so that insolvency is a rare event. For the sake of discussion (and subsequent simulations), let us suppose that the confidence level is set at 0.95 over a $10-$ year horizon. That is, the Insurance fund size is set so that the probability of its being depleted over any $10-$ year horizon is 5 percent.

The dynamics of the FDIC's insurance fund are as follows. Denote by C_n the size of the fund, in billions of dollars, at the start of period n. The cashflows to the fund are the premiums κ_n (determined from expression (1)), the losses L_n, and any rebates $R_n \in (-\infty, \infty)$ necessary. If losses are smaller than anticipated, then the rebate is positive. However, if losses are greater than expected, then the rebate will be negative. A negative premium is a replenishment of the fund by the banking industry. The rebates occur at the end of the period.

Let p represent the target probability (e.g. 0.05) that the Insurance fund is depleted over a fixed $T-$period horizon (e.g. 10 years). Let the initial size of the fund be $C_0 = C$, contributed by the banking industry. The fund size at the start of the next period $(n \geq 1)$ is then given by

$$C_{n+1} = C_n(1 + r_n) + \kappa_n(1 + r_n) - L_n - R_n. \tag{2}$$

At time n, given that both C_n and κ_n are fixed, and L_n is outside the control of the FDIC, the rebate R_n determines the probability of insolvency over the next time period, i.e. R_n determines $\mathbf{prob}_n(C_{n+1} \leq 0)$ where $prob_n(\cdot)$ is the statistical probability conditioned on the information available at time n. The use of expressions (1) and (2) makes explicit this probability's dependence on the rebate, i.e.

$$\mathbf{prob}_n(C_{n+1} \leq 0) = \mathbf{prob}_n(C_n(1+r_n) + \widetilde{E}_n(L_n) - L_n - R_n \leq 0). \tag{3}$$

This is intuitive. The rebate determines the funds balance at time n, $(C_n(1+r_n) - R_n)$, from which the unexpected losses, $(\widetilde{E}_n(L_n) - L_n)$, are absorbed.

The decision problem for the FDIC at time 0 is to choose the state contingent rebates[2] $\{R_1, ..., R_T\}$ such that

$$\mathbf{prob}(\min\{C_i : i = 1, ..., T\} \leq 0) = p. \tag{4}$$

At the beginning of each period, the rebate must be chosen (based on the available information) so as to keep the insolvency of the fund over the $T-$ period horizon equal to p. Note that this is the only rebate and premium structure that satisfies the two conditions: (i) the premium is "fair" in the sense of expression (1), and (ii) the fund size satisfies the p probability insolvency constraint (4) at time 0. No other fair premium and rebate system satisfies these two constraints. This observation will prove important below. As written, we see that the determination of the $T-$ horizon rebates is inherently a multiperiod problem. This is because the probability p represents a first passage probability.

To gain some intuition into the solution to this problem, for illustration purposes only, we consider the special situation where the events $\{C_n > 0\}$ for $n = 1, ..., T$ are assumed to be independent and identically distributed under the statistical probability measure. Under this

[2] The rebates $\{R_n\}$ are measurable with respect to the information available at time n.

i.i.d. hypothesis, let $\widetilde{p} \equiv prob\{C_n \leq 0\}$. Then

$$\begin{aligned}
\mathbf{prob}(\min\{C_n &:& n = 1, ..., T\} \leq 0) \quad (5) \\
&=& \mathbf{prob}(\textstyle\bigcup_{i=1}^{T} \{C_n \leq 0\}) \\
&=& 1 - \mathbf{prob}(\left[\textstyle\bigcup_{n=1}^{T} \{C_n \leq 0\}\right]^c) \\
&=& 1 - \mathbf{prob}(\textstyle\bigcap_{n=1}^{T} \{C_n > 0\}). \\
&=& 1 - \textstyle\prod_{n=1}^{T} prob\{C_n > 0\} \\
&=& 1 - (1 - \widetilde{p})^T.
\end{aligned}$$

In this special case, if we choose R_n such that

$$\mathbf{prob}(C_n(1 + r_n) + \widetilde{E}_n(L_n) - L_n - R_n \leq 0) = (1 - \sqrt[T]{(1 - p)}) \quad (6)$$

in each period, then this selection will give the solution to the FDIC decision problem (4). Indeed, by expressions (3) and (5) we have that

$$\mathbf{prob}(\min\{C_n : n = 1, ..., T\} \leq 0) = p.$$

In this special case, the multiperiod problem simplifies into a sequence of single period problems, where in each period the rebate is chosen so as to satisfy expression (6). This single period solution confirms the previous statement that this is the only rebate and premium structure that satisfies the two conditions: (i) the premium is "fair" in the sense of expression (1), and (ii) the fund size satisfies the p probability insolvency constraint (4) at time 0. Unfortunately, the assumption of independent and indentically distributed fund sizes C_n is, in general, inconsistent with expression (2)[3]. Consequently, solving the multiperiod problem first passage problem is

[3] To see this, note that with $r_n = R_n = 0$ for all n, we have $C_1 = C_0 + \kappa_1 - L_1$ and $C_2 = C_1 + \kappa_2 - L_2 = C_0 + \kappa_1 - L_1 + \kappa_2 - L_2$. We see that C_1 and C_2 having a common component will, in general, not be independent

essential for understanding the determination of the appropriate rebate.

Returning to the multiperiod setting again, an important property of the premium as determined by expression (1) is that the premiums change over time. This is due to the state of the economy changing (the conditional expectations) and the random spot rate of interest. For various reasons, the banking industry may desire a constant premium $\overline{\kappa}$ set for a fixed period of time, say T years. Unfortunately, if a constant premium is charged and added to the Insurance fund (as received), then either the premium will not be fair or the fund size will not satisfy the p probability insolvency constraint. To overcome this dilemma, the FDIC could enter into a T period cash flow swap (directly or synthetically) where it pays out the constant premium $\overline{\kappa}$ received, and it receives the fair premium and rebate necessary to maintain the funds p probability of insolvency $(\kappa_n(1+r_n) - R_n)$. The constant premium $\overline{\kappa}$ is determined so that the swap should be written at time 0 with no cash changing hands, i.e. at zero value. By including the rebate in the swap's cash flows, the premium would be neither pro- nor countercyclical.[4] Standard methods show that the premium $\overline{\kappa}$ should be set such that

$$\overline{\kappa} \sum_{n=0}^{T-1} B(0,n) = \sum_{n=0}^{T-1} \widetilde{E}\left(\frac{\kappa_n(1+r_n) - R_n}{b_n}\right) \tag{7}$$

where $B(0,n) = \widetilde{E}_0\left(\frac{1}{b_n}\right)$ is the price at time 0 of a sure dollar paid at time n and $b_n = (1 + r_0) \cdots (1+r_{n-1})$ is the time t value of a money market account growing at the spot rate of interest.

Using expression (1), this can equivalently be written as:

$$\overline{\kappa} = \frac{\sum_{n=0}^{T-1} \widetilde{E}\left(\frac{L_n - R_n}{b_n}\right)}{\sum_{n=0}^{T-1} B(0,n)}. \tag{8}$$

Under this system, the FDIC would charge the banks the constant premium, pay them out in the swap, and receive the correct cash flows per period to maintain the fund's solvency probability

and identically distributed.

[4] If one were to exchannge only $\kappa_n(1+r_n)$ then the constant premium plus the rebate would be pro-cyclic.

at the desired level. Note that the dynamics under this swap for the fund growth would still be expression (2), since the premiums $\overline{\kappa}$ would be replaced by the cash flows $(\kappa_n(1+r_n)-R_n)$.

Alternatively, instead of having the banking industry pay a constant premium, one could also design a countercyclical payment structure by entering into a T period swap of the cash flows $(\kappa_n(1+r_n)-R_n)$ - to receive- for the cash flows P_n - to be paid, at the end of each period. For example, let the annual premium assessed at the end of the year n be given by

$$P_n = \widehat{\kappa}\left(\max\left(\frac{C_n}{C},1\right)\right)^{-\beta}(1+L_n)^{-\gamma}. \tag{9}$$

As specified, the premium is set relative to a fund size of level C. The elasticity of the premium rebate with respect to C_n exceeding C is β. We also introduce a rebate with elasticity γ with respect to the aggregate loss level L_n. For $C_n < C$ and $L_n = 0$ the premium set by equation (9) is the flat rate of $\widehat{\kappa}$ billion dollars. Note that $\widehat{\kappa}$ will differ from $\overline{\kappa}$ determined in the previous section.

Similar to the situation before, if this premium is charged and added to the Insurance fund (as received), then either the premium will not be fair or the fund size will not satisfy the p probability insolvency constraint. The FDIC can still maintain these two constraints by entering into a different T period swap of the cash flows P_n out, for the cash flows $(\kappa_n(1+r_n)-R_n)$ in. The countercyclical premium parameters $(\widehat{\kappa},\beta,\gamma)$ would need to be set such that this swap has zero value, i.e.

$$\widehat{\kappa}\sum_{n=0}^{T-1}\widetilde{E}\left(\frac{\left(\max\left(\frac{C_n}{C},1\right)\right)^{-\beta}(1+L_n)^{-\gamma}}{b_n}\right)=\sum_{n=0}^{T-1}\widetilde{E}\left(\frac{\kappa_n(1+r_n)-R_n}{b_n}\right)$$

or (fixing (β,γ)):

$$\widehat{\kappa}=\frac{\sum_{n=0}^{T-1}\widetilde{E}\left(\frac{L_n-R_n}{b_n}\right)}{\sum_{n=0}^{T-1}\widetilde{E}\left(\frac{\left(\max\left(\frac{C_n}{C},1\right)\right)^{-\beta}(1+L_n)^{-\gamma}}{b_n}\right)}.$$

Note that the dynamics under this swap for the fund growth would still be expression (2), since the premiums P_n would be replaced by the cash flows $(\kappa_n(1+r_n) - R_n)$.

Alternatively, the FDIC could retain the premiums P_n and relax either the fair premium or the p probability insolvency constraint. This might be necessary if markets are incomplete with respect to bank failure losses, and where the cash flow swap is not attainable either directly or through synthetic construction. This is arguably the current situation in current US banking markets.

If we retain the p probability insolvency constraint, but relax the fair premium condition, then the insurance fund dynamics would grow according to the equations

$$C_{n+1} = C_n(1+r_n) + P_n - L_n \tag{10}$$

where the parameters $(\widehat{\kappa}, \beta, \gamma)$ are chosen such that

$$p = \mathbf{prob}(\min\{C_n(1+r_n) + P_n - L_n : n = 1, ..., T\} \leq 0). \tag{11}$$

Under these dynamics, the premium P_n is unfair to the extent that it differs from $(\kappa_n(1+r_n) - R_n)$. However, the fund size insolvency constraint is satisfied.

There are two approaches to pricing derivatives in an incomplete market. The first is to use some utility function or risk measure to select one of the risk neutral measures. This approach preserves the linearity of the pricing methodology. The second approach is useful for loss insurance contracts and other structured products. For these claims, their transaction prices are in a specified direction, akin to ask prices for the sale of cash flows. For such markets, there is an extensive literature on modeling the bid and ask prices. Since our focus here is on the ask price, we shall restrict our comments to it.

One model for the ask price is to use the cost of superreplication. Typically this price is too high and rarely will market ask prices be at such a level. Instead, Carr, Geman, and Madan

(2001) generalize superreplication to pricing by acceptability. Acceptability pricing requires that the residual cash flow from the price and the liquid asset hedge, less the payout of the liability, be in a cone larger than the positive orthant. This cone is referred to as the cone of acceptable risks and satisfies the axioms for acceptable risks described in Artzner, Eber, Delbaen and Heath (1998). Such ask prices are given by a nonlinear and concave functional of the cash flows. The ask price is also the supremum over all expectations (associated with an admissible collection of risk neutral measures) of the discounted cash flows. In this regard, equation (1) continues to hold, but now the measure used is not independent of the cash flow being priced. The measure will vary with the description of the cone of acceptable risks, but will be independent of the size of the FDIC capital insurance fund, as was the case in the previous context. For further details on these issues we refer the reader to Eberlein and Madan (2006).

This paper does not construct the fair market prices that would prevail in either a complete market or incomplete context. Instead, we study the joint structure of capital level and premia systems that meet a $10-$ year first passage survival probability target. An analysis of such countercyclical capital and premium system tradeoffs is provided in the next section. A full solution would require the identification of the ask price that reduces under replication to the risk neutral price. The capital is then at the level appropriate for this premium from among the tradeoffs we describe.

3 Countercyclical Premium System

To simplify the analysis, we assume that the economy is in steady state, where the deposit growth rate equals the spot rate of interest in the economy. Then, the premiums, losses and fund size all grow at the same rate at which discounting occurs. This implies that in computing present values, the discount rates cancel. This is equivalent to setting the interest rate equal to zero in the economy discussed in the previous section. We study the countercyclical insurance premium computations in this steady state economy. We suppose the system starts out with the benchmark level for the fund size of C, and we present the trade-offs in the flat rate $\widehat{\kappa}$, the rebate elasticities β, γ that are consistent with long term fund survival measured by a 95% target probability of surviving 10 years.

The generation of annual aggregate losses L_n and the operation of the system over time are as follows. We analyze the long term survival probabilities by simulating the annual losses for ten years. A random number M_n of failures each year generate the aggregate loss amount. Each of these failures has an associated asset size, A_k for failure by bank k, and loss rate l_k with the k^{th} loss amount being $A_k l_k$ and $L_n = \sum_{k=1}^{M_n} A_k l_k$. The asset sizes are drawn from a stable aggregate distribution of asset sizes, and likewise loss rates are drawn from a stable and independent distribution of loss rates. The loss arrivals are generated by a Poisson process with a constant arrival rate.

The asset sizes that are relevant are those of all banks that are subject to failure in any given year. The loss rates are those that have been experienced in past bank failures. For the US banking system, the distribution of asset sizes has a characteristic property of a substantial number of banks that have assets that are an order of magnitude above the model or most likely asset size. Such a fat tailed distribution can be well modeled by the parametric class of Frechet distributions. This distribution has two parameters, a scale parameter c_F and a shape parameter a_F with the cumulative distribution function $F(A; c_F, a_F)$ given by

$$F(A; c_F, a_F) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right). \tag{12}$$

The associated density is $f(A; c_F, a_F)$ and

$$f(A; c_F, a_F) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right) \frac{a_F c_F^{a_F}}{A^{1+a_F}} \tag{13}$$

and the tail of the distribution falls at rate $1 + a_F$ with the consequence that moments exist only for orders less than a_F. Hence the mean, μ_F, is finite for $a_F > 1$ and the variance, σ_F^2 is finite for $a_F > 2$ in which case

$$\mu_F = c_F \Gamma\left(1 - \frac{1}{a_F}\right) \tag{14}$$

$$\sigma_F^2 = c_F^2 \left(\Gamma\left(1 - \frac{2}{a_F}\right) - \Gamma\left(1 - \frac{1}{a_F}\right)^2\right) \tag{15}$$

The Frechet distribution has a mode A_m below c_F at the point

$$A_m = c_F \left(1 + \frac{1}{a_F}\right)^{-\frac{1}{a_F}} \tag{16}$$

that reflects a positive most likely asset size and yet it has a long tail with substantial probability at large sizes as the density decays at a power law. In this regard it is particularly suited for describing the distribution of asset sizes in a population of many small banks with a few very large ones.

For the loss rate distribution we consider the parametric class of the Weibull distribution. Loss rates are essentially bounded variables for which all moments are finite. This is true for the Weibull family. This family also has two parameters, a scale parameter c_W,and a shape parameter a_W with the cumulative distribution function $G(L; c_W, a_W)$ given by

$$G(L; c_W, a_W) = 1 - \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right). \tag{17}$$

The associated density is $g(L; c_W, a_W)$ and

$$g(L; c_W, a_W) = \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right) \frac{a_w L^{a_W - 1}}{c_W^{a_W}} \tag{18}$$

The mean, μ_W and variance, σ_W^2 are given by

$$\mu_W = c_W \Gamma\left(1+\frac{1}{a_W}\right) \tag{19}$$

$$\sigma_W^2 = c_W^2\left(\Gamma\left(1+\frac{2}{a_W}\right)-\Gamma\left(1+\frac{1}{a_W}\right)^2\right) \tag{20}$$

For $a_W < 1$ this density has a mode at zero representing a most likely loss rate of zero. However, for the case $a_W > 1$ we have a modal loss level L_m below c_W of

$$L_m = c_W\left(1-\frac{1}{a_W}\right)^{\frac{1}{a_W}}. \tag{21}$$

Furthermore, in this case the probability in the upper tail decreases at a rate that is faster than exponential which makes loss rates near unity relatively uncommon. The shape parameter of the Weibull distribution, a_W parametrizes the behavior of the hazard rate for losses. The hazard rate is the relative probability of a large loss rate to an even larger loss rate. When hazard rates are increasing it gets more and more difficult to get to higher and higher loss rate levels. For the Weibull, with $a_W < 1$ we have decreasing hazard rates while for $a_W > 1$ we have increasing hazard rates.

For the distribution of loss rates we anticipate a positive mode, with an increasing hazard rate as all attempts are being made to limit losses. Hence the Weibull model with shape parameter above unity is appropriate. We also note that the Weibull model would be inappropriate for asset sizes as it has a fat tail only when its mode is zero while asset sizes have a fat tail with a positive mode. Similarly, the Frechet model is inappropriate for loss rates as it would generate a large number of loss rates above unity. Our empirical tests on the data confirm these conjectures.

Other distributional candidates may also be considered from the prior literature (Madan and Unal, 2004; Unal, Madan, and Guntay (2004), Kuritzkes, Schuermann, and Weiner, 2004). We also provide additional empirical tests of these alternatives with respect to our proposed choices and confirm the adequacy of the model we adopt.

3.1 Empirical Tests of the Distributional Models

For the distribution of asset sizes we use the asset sizes of 8694 banks in the US from the Call Report Data for the year 2000. The loss rate data come from the failed bank data base maintained at the FDIC for the period $1984-2000$. The number of failures were 1505 of which 32 had a zero loss rate. Our analysis uses the 1473 failed banks with positive loss rates. Table 1 summarizes the time series loss experience of the FDIC for the $1984-2000$ period. It shows the yearly estimated losses as a percentage of the total assets of the failed banks together with the number of failures for six size categories. Three trends are observable. First, number of bank failures decline as bank asset size increases. For example, only eight banks over $5 billion asset size failed during the sample period. Second, as asset size increases loss rates decline. For example, while average loss rate for the smallest asset size group is about 25%, for the largest size group this average percentage declines to about 8%. Finally, after 1992, there is a notable decline in the number of bank failures.

The summary statistics on asset sizes and loss rates on failed banks are reported in Table 2. We observe that for the asset size the mean is substantially above the median and in fact also above the upper quartile, suggestive of a highly skewed and fat tailed distribution. This property is also reflected in the large standard deviation. Furthermore the last percentile relative is 51 times the upper quartile. Hence, the Frechet distribution appears to be an appropriate choice.

We observe that the average loss rate for the $1984-2000$ period is 21.1% of the assets. The mean and median loss rates are fairly close with the mean in the interquartile range. In addition, the last percentile is well below the unit loss rate and this observation suggests a substantially thinner upper tail. Thus, the Weibull model with a shape parameter above unity, appears a reasonable choice. The differences between the loss rate and asset size data sets are quite marked and provides the early indication that it is not likely that the two data sets come from the same distributional model.

We estimate both the Frechet (equation 13) and Weibull (equation 18) models by maximum likelihood on the asset size data scaled to \$10 billions, for the 8649 banks in the year 2000. The parameter estimates for the Frechet are $a_F = 0.94002$, $c_F = 0.005154$ and the estimates for the Weibull are $a_W = 0.5426$, $c_W = 0.0204$. For graphical convenience Figure (1) plots the histogram of the binned data in steps of \$10 million up to \$500 million. This segment contains 90% of the data.

We observe that the Frechet model fits the data better. It picks up the mode and the long tail quite accurately. The Weibull on the other hand tries to get the long tail and as a consequence is forced to place the mode at zero. The quality of the improvement of the Frechet over the Weibull model is confirmed by Chi Square tests performed in the range of cells with more than 10 observations that go up to asset sizes of \$700 million. The Frechet model could be improved upon in the smaller asset sizes below \$250 million. For the range from \$250 million to \$700 million we have 43 degrees of freedom with the Frechet chi square statistic of 56.69 while the corresponding Weibull value is 416.67. The respective $p - values$ are .0787 for the Frechet and zero for the Weibull.

For the loss rate data, in addition to the Frechet and Weibull models, we employ three other distributions that have been used to describe loss distributions in the literature (Madan and Unal, 2004; Unal, Madan, and Guntay (2004), Kuritzkes, Schuermann, and Weiner, 2004). These are the Gaussian with parameters μ_G, σ_G, the Beta distribution with two parameters α, β and the logitnormal with parameters μ_L, σ_L. The results are presented in Table 3.

Table 3 shows that the Beta and Weibull models dominate the Gaussian, Frechet and Logit Normal as candidates for this distribution. The Weibull reflects a mode and an increasing hazard rate with a fit that is marginally better than the Beta distribution. Figure (2) presents a graph of the histogram of loss rates and the fitted distributions.

We observe from figure (2) the relative closeness of the Weibull and Beta model to each other and the data (displayed as circles). The Gaussian model comes next followed by the Logit normal and the Frechet. This visual ranking of the models is formally confirmed in the χ^2 statistics and corresponding $p - values$. For the latter we use 50 bins with more than 5 observations with the resulting degrees of freedom being 48. The test statistics reported delete the bottom 10% of loss rates and hence we have 38 degrees of freedom.

4 Simulation Results

4.1 Design

A typical simulation run of traces the annual progression of the aggregate premium, loss levels and the fund size at beginning of each year for 10 years for 1000 potential paths. The run produces three 10 by 1000 matrices for the aggregate premium, annual loss level and beginning of year fund size. Equation (9) defines the annual premiums. The aggregate annual loss amount is generated by simulating a Poisson number of failures with mean arrival rate of $\lambda = 20$, for each of which we simulate an asset size from the estimated Frechet distribution, and a loss rate from the estimated Weibull distribution, with the aggregate annual loss being sum over the number of losses of the product of the asset sizes and loss rates. The initial fund size for the next year is defined by equation (**??**) using the premiums and losses that were generated for the year. On any path for which the funds size reaches the bankruptcy level at the start of some year, the simulation on this path is stopped with the fund size frozen at the bankruptcy level. For the default probability we count the proportion of bankrupt states in the 1000 paths.

For the three components of the loss simulation, the number of failures, the associated asset size and loss rate the details are as follows. For the Poisson number of losses we use the Poisson random number generator from Matlab and generate N_{nm} the number of failures in year n on path m with a constant arrival rate of $\lambda = 20$. This assumption of mean failure rate draws on the failure experience of the FDIC during the post FDICIA period.

For each failure $i \leq N_{nm}$, the asset size we generate a uniform random number $u_{nm}^{(i)}$ for year n on path m and simulate the asset size $A_{nm}^{(i)}$ in accordance with the inverse cumulative distribution method,

$$A_{nm}^{(i)} = c_F \left(-\ln \left(u_{nm}^{(i)} \right) \right)^{-\frac{1}{a_F}}. \tag{22}$$

where, $\alpha_F = 0.94, c_F = 0.0051$. To prevent the asset size reaching unreasonable levels we introduce asset cap, p, and modify equation (22) as follows

$$A_{nm}^{(i)} = c_F \left(\left(\frac{c}{p} \right)^{\alpha_F} - \ln \left(u_{nm}^{(i)} \right) \right)^{-\frac{1}{a_F}}. \tag{23}$$

We set p equal to \$500 billion, which is roughly the largest asset size insured bank in the United States in the base year of 2000.

Similarly, for the loss rate associated with failure i , $l_{nm}^{(i)}$ for year n on path m we generate another independent sequence of uniform random variates $v_{nm}^{(i)}$ with the loss rates now given by the inverse Weibull cumulative distribution function

$$l_{nm}^{(i)} = c_W \left(-\ln \left(1 - v_{nm}^{(i)} \right) \right)^{\frac{1}{a_W}}. \tag{24}$$

where, $\alpha_W = 1.7031, c_W = 0.2404$.

The aggregate annual loss amount for year n on path m, L_{nm} is then

$$L_{nm} = \sum_{i=1}^{N_{nm}} A_{nm}^{(i)} l_{nm}^{(i)}. \tag{25}$$

The fund size at the start of year n on path m is C_{nm}. The loss amount for the year is L_{nm}. The premium for the year on this path is

$$P_{nm} = \widehat{\kappa} \left(\max \left(\frac{C_{nm}}{C}, 1 \right) \right)^{-\beta} (1 + L_{nm})^{-\gamma} \tag{26}$$

where the policy parameters $\widehat{\kappa}, \beta, \gamma, C$ are prespecified. The fund size at the start of the next year is then

$$C_{n+1,m} = C_{nm} + P_{nm} - L_{nm}. \tag{27}$$

The simulation on a path is stopped the first time C_{n+1}, m is below the bankruptcy threshold of half a billion dollars. We assume that the interest earned on the fund balance equals to the expenses of running the insurance fund.

4.2 The Current State

Table 4 reports results for a number of base case alternatives where we assume no rebates are given and a flat premium structure to exist. Case 1 shows that we assume the starting fund size to be 31 billion dollars with total domestic deposits of 3.3 trillion dollars. In addition, we assume premium income to be zero. These initial assumptions are roughly consistent with the state of the insurance fund at FDIC during early 2000s. For a flat premium structure with no countercyclical features β and γ are zero in equation (9). For this setting of the simulation inputs, given the simulated losses, and assuming no premiums are paid during the ten-year period, we find that the default probability in 10 years is 19%.

For a target default probability in 10 years of 5% with no countercyclical features, one may adjust upward either the fund size or the aggregate premium level. Case 2 shows that keeping the premium level at zero, it takes about doubling of the fund size to \$62.5 billion to reduce the 10 year default probability below 5%. Alternatively Case 3 demonstrates that keeping the fund size at 31 billion dollars one may raise the level of premiums to \$5 billion per year (or an effective assessment rate or .15%) to reduce this 10 year default probability below 5%. Finally, Case 4 shows an intermediate possibility where the fund reserve is raised to 40 billion and the effective assessment rate is increased to .078% to attain the target 5% default probability.

The above simulations employ the same aggregate loss distribution over the ten years as the random number seed is fixed. This distribution is made up of three components cumulated over ten years, and these are the Poisson arrivals, the Frechet assets sizes capped at \$500 billion, and Weibull loss rates each year.

4.3 Countercyclical Trade-offs

We next explore the trade-offs inherent in the design of countercyclical premium systems in Table 5. We consider a number of sample premium schedules around the base scenario of Case 4 in (C_0 = \$40 billion fund size and a flat premium level κ = \$2.6 billion or an effective assessment rate of (EAR) \$2.6 billion/\$3.3 trillion = 0.078% that gives a 10−year 5% default probability).

Panel A introduces the base case when no rebates are considered. Panel B introduces rebates based on the level of aggregate losses only. In other words, in terms of Equation (9), we assume $\beta = 0$. In this case, to organize loss rebates the insuring agency needs to decide on the parameter value of γ. Suppose it calls it to be 3.802.This value determines the discount that will be applied to the zero-rebate premium:

$$P_n = \widehat{\kappa}(1 + L_n)^{-3.802}. \tag{28}$$

Every year, depending on the losses on bank failures (L_n),the insuring agency reduces the κ and assesses the premium level. In a given year if there are no losses the banks pay \$$\widehat{\kappa}$.If losses are high their premium level is lower.

The parameter value 3.802 implies the following in terms of zero rebate premium. The insuring agency allows a 50% reduction in the annual \$2.6 billion premium level when aggregate losses are \$2 billion. The value of γ that satisfies the equation ($50\% = (1+L)^{-\gamma}$), where $L = .2$, accomplishes this premium reduction.[5] Thus $\gamma = 3.802$.

Table 6 Panel B, Column 9 shows that such rebate structure increases the default probability of the fund from 5% to 7.3%. Note that banks will pay into the system the 0.078% EAR annually

[5] We express \$5 billion as .5 because our calculations are in terms of 10s of billion dollars.

if there are no losses over the next ten years. However, in this case we allow for losses to be incurred and rebates to be given as a result of costly bank failures. Therefore, the actual EAR varies annually because it is indexed to the aggregate loss levels and on average they should be less than 0.078%. Column 6 shows that on average the EAR is 0.048% with a standard deviation of 0.01%. This reduction in premium payments results in an increase in the default probability of the system.

We now ask what the effect is on the flat rate premium of introducing such a rebate if we wish to bring back the default probability back to 5%. Table 5 shows that the flat rate premium associated with a zero level of losses ($\widehat{\kappa}$) with loss-rebate structure in place, now rises to $6 billion or an effective assessment rate of 0.182%, to maintain the target 10 year default probability at 5%.

Alternatively, the policy choice for γ could be set at a higher level than 3.802 to give rebates at a higher rate. Table 6 shows two such choices where γ is set to be 14.207 and 7.273. These values imply that the insuring agency pays rebates when the loss on failures accumulates to $0.5 billion and $1 billion, respectively. In either case default probabilities increase from 7.3% to 9.1% and 8.5%.

One consequence of the loss-rebate only system is that the fund size can keep growing over time to ensure a 5% default probability over ten years. We can adjust the premium structure such that the fund level's growth is slowed down but at the same time the target default probability remains unchanged. One strategy is to collect higher premiums in early years leading to lower levels of the fund size in later years. Such design can be accomplished by augmenting the loss rebate with a rebate system associated with the size of the fund. For this purpose the insuring agency needs to decide on the parameter β. Suppose $\beta = 4.122$ is chosen and no loss rebates are allowed. Then the discount applied to zero capital (and zero loss) premium is

$$P_n = \widehat{\kappa} \left(\frac{C_n}{C} \right)^{-4.122} . \tag{29}$$

Panel C shows that when such a structure is in effect the default probability increases to 5.7%. An increase in the zero capital rebate level from \$2.6 billion to \$4 billion reduces the default probability back to 5%.

The parameter value 4.122 implies the following in terms of the starting fund size of \$40 billion. If the fund size rises to \$45 billion, a 12.5% increase, there will be a 38.46% rebate in the \$2.6 billion premium (the \$5 billion excess fund size will be returned to the banking sector roughly in five years). Hence the premium associated with an excess fund size of 12.5% is \$1.6 billion. Such a rebate is organized by $\beta = 4.122$ ($.16 = .26 * \left(\frac{45}{40}\right)^{-\beta}$).

The remainder of the table shows more conservative levels of capital rebates. Finally, Panel C brings together both capital and loss rebates and identifies the default probabilities and premium structure.

Tables 4 and 5 underscore an important fact about countercyclical deposit insurance system. For the system to enjoy the luxury of the rebates needed for countercyclical deposit insurance pricing, either the insurance fund must have a super surplus or the premium structure need to be increased drastically. For example, In Table 6 we observe that a loss and fund-size rebate system requires the EAR to increase from 0.078% (no rebate system EAR) to 0.107%. At a level of \$3.3 trillion insured deposits, this EAR implies that the banking system needs to pay an additional \$.9 billion on average per year over a ten-year period to enjoy the benefits of a countercyclical premium system. Given that the EAR for the no rebate system is 0.078%, this additional premium represents an average of 35% increase in aggregate premium level.

The limitations of these findings are as follows. First, the loss distribution we use is estimated using the FDIC experience during the 1984-2000 period. We realize that this period may not reflect the loss distribution faced by the FDIC in the next decade. One important consideration is the prompt corrective action (PCA) provision of the FDICIA, which requires regulatory intervention in advance of insolvency. Such mandate can substantially reduce expected costs (Blinder and Wescott, 2001). However, we use this period to allow for the possibility of adverse macro shocks experienced in the 1980s.

Second, we assume the same loss rate distribution for large and small bank failures. However, Bennett (2000) shows that cost of resolving small banks is much higher than the cost of resolving large banks. Hence, loss rates and failed bank asset size are correlated. Our simulations do not allow for such association and apply the same loss rate distribution for small and large bank failures.

Therefore, our results should be taken as the upper limit for a risk-based and countercyclical insurance premium structure that attains a target 95% survival probability of the insurance fund over ten years.

5 Conclusion

This paper provides a mechanism for exploring premium systems that are both responsive to relief in times of crisis and build in a distribution of excess fund sizes while preserving a risk based structure for determining aggregate premiums that ensure viability of the deposit insurance system. For this purpose we study the distributions of assets sizes and loss rates and determine that these are modeled well by the Frechet and Weibull families respectively. The paper shows that the benefits of a countercyclical rebate system does not come free. The system should be ready to pay a substantial cost to finance a countercyclical rebate system.

6 References

Allen, L. and A. Saunders, 2004, Incorporating Systemic Influences into Risk Measurements: A Survey of the Literature. Journal of Financial Services Research, V. 26, No. 2.

Atlan, M., H. Geman, D. Madan and M. Yor 2006, Correlation and the Pricing of Risks. Annals of Finance, forthcoming.

Artzner, P., F. Delbaen, J. Eber, and D. Heath, 1998, Definition of coherent measures of risk, Mathematical Finance 9, 3, 203-228.

Bennett, R. L., 2000. Loss Rates and Asset Size. Internal Memo, FDIC.

Blinder, A. S. and R. F. Wescott, 2001, "Reform of Deposit Insurance: A Report to the FDIC," http://www.fdic.gov/deposit/insurance/initiative/reform.html.

Carr, P., H. Geman and D. Madan, 2001, Pricing and Hedging in Incomplete Markets, Journal of Financial Economics, 62, 131-167.

Eberlein, E. and D. Madan 2006, Sato Processes and the Valuation of Structured Products, working paper, Robert H. Smith School of Business, University of Maryland.

Gordy, M.B. and B. Howells, 2004. Procyclicality in Basel II: Can We Treat the Disease Without Killing the Patient?

Madan, D. B. and H. Unal, 2004, "Risk-neutralizing Statistical Distributions: With an application to pricing reinsurance contracts on FDIC losses." Working Paper. Smith School of Business, University of Maryland.

Unal, H., D. Madan, and L. Guntay, 2004."A simple approach to estimate recovery rates with APR Violation from Debt Spreads," Journal of Banking and Finance.

Kashyap, A. K. and Stein, J. C., 2004, The Cyclical Implications of the Basel II Capital Standards, Federal Reserve Bank of Chicago Economic Perspectives 28, 18-31.

Kuritzkes, A., T. Schuermann, and S. M. Weiner, 2004, "Deposit Insurance and Risk Management of the U.S. Banking System: What is the loss distribution faced by the FDIC?, working paper. Federal Reserve Bank of New York.

Kusuoka, S. (2001) On law invariant coherent risk measures. In Adavances in Mathematical Economics, volume 3, pp. 83-95. Springer. Tokyo.

Pennacchi, G. G., 1999. The Effects of Setting Deposit Insurance Premiums to Target Insurance Fund Reserves, Journal of Financial Services Research, 16 (2/3), 153-80.

Pennacchi, G. G., "Risk-Based Capital Standards, Deposit Insurance, and Procyclicality, forthcoming, Journal of Financial Intermediation.

White, W. R., 2006. Procyclicality in the Financial System: do we need a new macrofinancial stabilization framework?. BIS Working Paper, No. 193.

Table 1: Total assets, loss as a % of total assets, number of bank failures

	Over 5 B	1-5 B	500M-1B	100-500M	50-100M	Under 50M
1984	39,957(7%)	-	513(1%)	1,345(13%)	419(16%)	1,197(23%)
	1	-	1	7	7	64
1985	5,279(7%)			1,075(9%)	454(22%)	1,928(25%)
	1	–	–	5	6	108
1986	–	1,589(14%)	598(23%)	1820(26%)	1468(25%)	2164(27%)
		1	1	10	21	112
1987	–	1,200(0%)	501(13%)	3,284(26%)	1,251(24%)	2,993(27%)
		1	1	15	18	168
1988	18,162(11%)	10,949(13%)	7,717(9%)	10,788(12%)	3,560(15%)	3,280(26%)
	1	4	12	53	51	159
1989	7,181(22%)	6,932(22%)	4,373(16%)	8,739(14%)	1,685(23%)	2,695(25%)
	1	3	6	37	25	135
1990	–	4,144(9%)	1,950(12%)	5,703(24%)	1,488(19%)	2,455(20%)
		2	3	26	22	116
1991	45,591(3%)	9,146(16%)	4,619(22%)	5,943(23%)	1,535(18%)	1,629(21%)
	4	7	6	23	21	66
1992	7,269(10%)	23,704(5%)	3,421(15%)	8,304(10%)	1,456(18%)	1,334(18%)
	1	9	6	32	20	54
1993	–		936(13%)	1,389(20%)	582(21%)	621(20%)
			1	7	8	25
1994	–	–	–	1,217(12%)	77(23%)	111(10%)
				7	1	5
1995	–	–	–	635(10%)	77(13%)	31(29%)
				3	1	2
1996	–	–	–	–	114(19%)	68(25%)
					2	3
1997	–	–	–	–	–	26(19%)
						1
1998	–	–	–	375(60%)	–	53(8%)
				1		2
1999	–	614(0%)	–	115(9%)	157(27%)	61(10%)
		1		1	2	3
2000	–	–	–	114(11%)	239(6%)	38(5%)
				1	3	2

Table 2: Asset Size and Loss Rate Summary Statistics

	Asset Size ($Billions)	Loss Rate (%)
Mean	.751	21.10
Standard Deviation	10.0	12.97
Minimum	.0013	.0053
Maximum	584	93.94
Median	.084	19.63
Lower Quartile	.042	11.64
Upper Quartile	.188	28.80
First Percentile	.008	.3084
Last Percentile	9.67	56.42

Table 3: Results on distributional models for loss rates

Model	Parameter 1	Parameter 2	χ_2	p-value (df=38)
Gaussian	$\mu_G = 0.2066$	$\sigma = 0.1319$	76.06	0.00024
Beta	$\alpha = 1.8454$	$\beta = 6.7546$	49.86	0.0942
Weibull	$\alpha_W = 1.7031$	$C_W = 0.2404$	48.13	0.1256
Frechet	$\alpha_F = 0.9814$	$C_F = 0.109$	570.46	0
Logit Normal	$\mu = -1.5182$	$\Sigma = 0.7175$	122.17	0

Table 4: Base case alternatives with no rebate system

	Case 1	Case 2	Case 3	Case 4
Fund size ($billion)	31	62.5	31	40
Domestic deposits ($trillion)	3.3	3.3	3.3	3.3
Effective assessment rate (%)	0	0	0.15	0.078
Aggregate premium ($billion)	0	0	5	2.6
10-year default probability (%)	19	5	5	5

Table 5: Trade-offs in the design of countercyclical premium system

	γ	β	LR	CR	κ	Zero Rebate EAR	Avg. Eff. EAR	EAR σ	DP (%)
Panel A									
	0	0	0	0	2.6	0.078	0.078	0	5
Panel B									
	3.802	0	2	0	2.6	0.078	0.048	0.01	7.3
	3.802	0	2	0	6	0.182	0.112	0.02	5
	14.207	0	.5	0	2.6	0.078	0.021	0.01	9.1
	14.207	0	.5	0	15	0.455	0.122	0.03	5
	7.273	0	1	0	2.6	0.078	0.035	0.01	8.5
	7.273	0	1	0	9	0.273	0.123	0.03	5
Panel C									
	0	4.122	0	1	2.6	0.078	0.061	0.01	5.7
	0	4.122	0	1	4	0.121	0.078	0.02	5
	0	1.8132	0	0.5	2.6	0.078	0.068	0.01	5.3
	0	1.8132	0	0.5	3.0	0.092	0.075	0.01	5
	0	1.2275	0	0.25	2.6	0.078	0.07	0.01	5.2
	0	1.2275	0	0.25	2.8	0.085	0.074	0.01	5
Panel D									
	7.273	1.813	1	0.5	2.6	0.078	0.034	0.01	8.5
	7.273	1.813	1	0.5	11	0.33	0.107	0.02	5

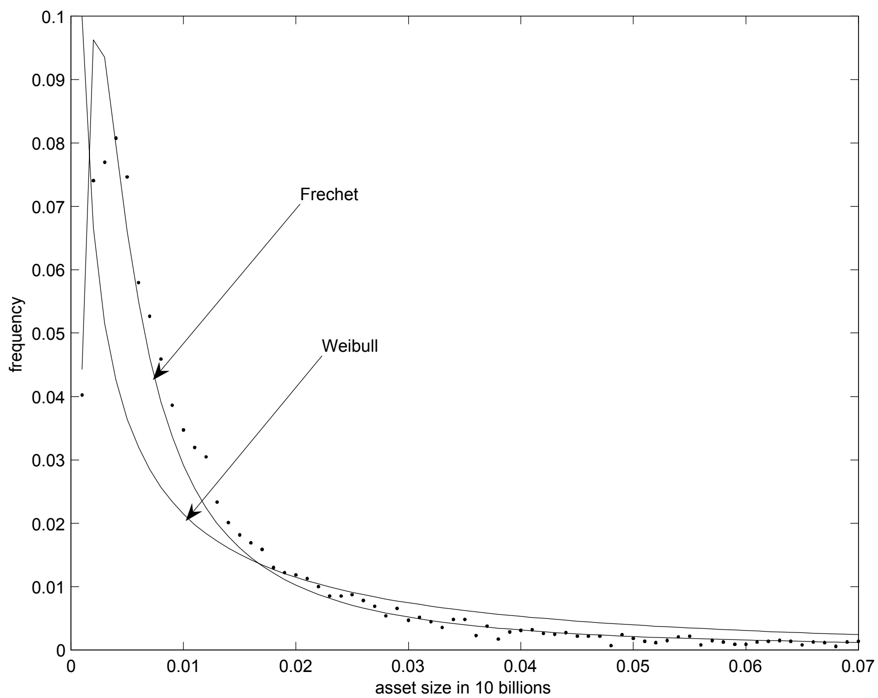

Figure 1: Asset Size Distributions on the Frechet and Weibull Models.

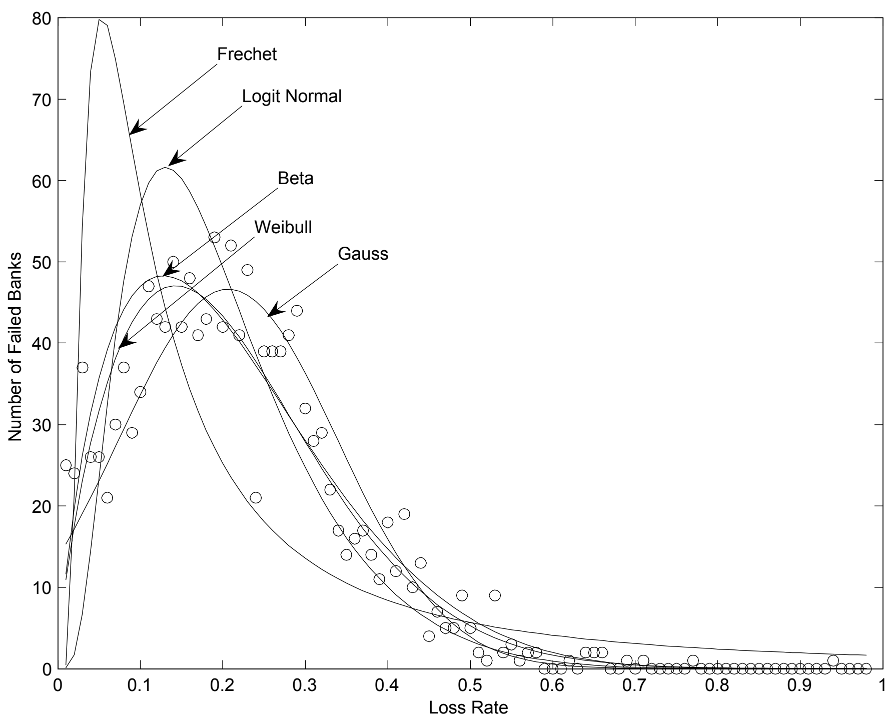

Figure 2: Loss Rate Distributions on the Gaussian, Beta, Weibull, Frechet and Logit Normal Models